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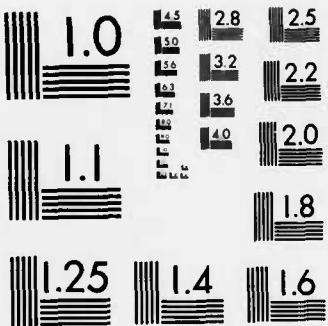
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A PROGRAM OF ELLIPTIC SOLVER DEVELOPMENT AND IMPLEMENTATION IN
SEMI-IMPLICIT NUMERICAL OCEAN CIRCULATION MODELS

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PREFACE

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1. INTRODUCTION

Beginning with the simultaneous discovery of the fundamental algorithm by Hirota, et al. (1970), and by Roache (1971), a new class of algorithms for solving elliptic partial difference equations has been developed during the last 10 years. Roache (1971) noted that the fundamental algorithm, which is analogous to "shooting methods" in ordinary differential equations, and described as the "error vector propagation" (EVP) method, is very efficient when applied to low-resolution problems, but cannot be applied to high-resolution problems due to roundoff error. Dietrich (1974) discovered an iterative application of the algorithm that could be applied to high-resolution problems, and this iterative technique was further explored and refined by Dietrich, et al. (1975) and by Dietrich (1977). Madala (1978) discovered a method, called the "stabilized error vector propagation" (SEVP) technique, whereby the basic algorithm can be applied as a direct method for high-resolution problems. Roache (1978) reviewed the development of EVP and analyzed its characteristics in detail.

One important class of applications of EVP-based elliptic solvers is in semi-implicit formulations of the fluid dynamic equations in which dominant

I. Hirota, T. Tokioka, and M. Nishiguchi, J. Meteor. Soc. Japan 48 (1970), pp. 161-167.

P. J. Roache, "A New Direct Method for the Discretized Poisson Equation," in Proceedings of the Second International Conference of Numerical Methods in Fluid Dynamics, Berkeley, California, September 16-19, 1970, pp. 48-53, 1971.

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R. Madala, Mon. Wea. Rev. 106 (1978), pp. 1735-1741.

P. J. Roache, Num. Heat Trans. 1 (1978), pp. 1-201.

terms involved in gravity (O'Brien and Hurlburt, 1972) and/or compression wave (Dietrich, et al., 1981) propagation are solved implicitly (with the remaining terms explicit). A more powerful application exists in solving "almost fully implicit" formulations of the fluid dynamic equations (Dietrich, 1975); this application is recommended when the shortest fluid dynamic time scale of interest is long, compared to the well-known CFL time step limit, as appears to be the case in most fluid dynamic problems with constant or slowly varying external forcing. (In some problems with waves generated by time-varying external conditions, this might not be the case.)

In Section 2 of this report, we describe an SEVP algorithm, program JAYNOR, for solving general two-dimensional, second-order elliptic equations. This algorithm can be applied to linear problems with variable coefficients, irregular boundaries, islands, and general boundary conditions. Its limitations are mainly associated with roundoff error, as described in Section 3. In Section 4, we describe how to use a computer program based on this general algorithm, as well as more specialized versions of this program. Appendix II contains listings of this general program and its more specialized versions.

J. J. O'Brien and H. E. Hurlburt, *J. Phys. Oceanogr.* 2 (1972), pp. 14-26.

D. Dietrich, H. Klein, and R. K.-C. Chan, "FLAME: A Computer Model of Time-Dependent, Multidimensional, Multiphase Reactive Flow," paper presented at the 20th National Heat Transfer Conference, Milwaukee, Wisconsin, August 2-5, 1981.

D. Dietrich, *J. Meteor. Soc. Japan* 53 (1975), pp. 222-225.

2. THE GENERAL SEVP ALGORITHM

The general SEVP algorithm described here is similar to the one described by Madala (1978). The differences are mainly the capability of including islands in the present version, and the elimination of the homogeneous solution calculation during the forward sweep by more fully utilizing influence coefficient arrays generated during a preprocessor.

The new SEVP algorithm, JAYNOR, uses a master flag array, $N_{i,j}$, defined by:

$$N_{i,j} = \begin{cases} 1 & \text{if the } (i,j) \text{ point is "inside the boundary"} \\ 0 & \text{otherwise} \end{cases}$$

Points on islands are considered "outside the boundary". If $N_{i,j} = 1$, the following equation is satisfied:

$$\begin{aligned} A_{i,j} \cdot X_{i,j+1} + B_{i,j} \cdot X_{i+2,j+1} + C_{i,j} \cdot X_{i+1,j} \\ + D_{i,j} \cdot X_{i+1,j+2} + E_{i,j} \cdot X_{i+1,j+1} = F_{i,j} , \end{aligned} \quad (1)$$

where $A_{i,j}$, $B_{i,j}$, $C_{i,j}$, $D_{i,j}$, and $E_{i,j}$ are given coefficient arrays of the five-point elliptic operator; $F_{i,j}$ is a given source term array; and $X_{i,j}$ is the solution array determined by JAYNOR. If $N_{i,j} = 0$, the $X_{i,j}$ is left unchanged. In addition to the above-indicated parameters and others normally needed to define a two-dimensional elliptic partial difference equation with second-order accuracy, JAYNOR requires the user to specify the number of subregions in which the aforementioned SEVP algorithm is to applied, and the grid lines separating these subregions. In general, the more subregions used, the smaller the roundoff error will be in the solution (at the expense of more auxiliary storage and calculation). These grid lines are transverse to the EVP march direction, which is the j -direction in JAYNOR.

We now describe the JAYNOR algorithm in detail. Since Neuman and mixed boundary conditions can be easily transformed into Dirichlet-type boundary conditions by suitably redefining the partial difference operator and

source term at points adjacent to boundaries (see Appendix I), we describe its application to problems with Dirichlet conditions. (JAYNOR assumes the user has already performed this simple task.)

First, we describe JAYNOR application to a regular region, illustrated in Figure 1. Two classes of auxiliary influence arrays are generated in the JAYNOR preprocessor, subroutine JAY. Both classes are composed of selected subvectors from the total solution vector, forced by a selected isolated unit source term. In a regular problem, each such subvector of the first class is the solution at the second row from the bottom of a subregion, forced by a unit source term assigned to a selected point in the second row from the top of the same subregion, in an associated problem in the region below the top boundary of the subregion — the associated problem has homogeneous boundary conditions, including at the top boundary of the subregion, and has zero source term everywhere, except at the selected unit source point. In JAYNOR, this array is called RINV; specifically, $RINV(I1, I2, K)$ is the solution at the $(I2+1)$ position of the second row from the bottom of subregion K, forced by an isolated unit source at the $(I1+1)$ position in the second row from the top of subregion K. In a regular problem, each subvector of the second class is the solution at the second row from the top of a subregion, forced by a unit source term assigned to a selected point in the second row from the top of the same subregion, again in the associated problem described above. In JAYNOR, this array is called RINV1; specifically, $RINV1(I1, I2, K)$ is the solution at the $(I2+1)$ position of the second row from the top of subregion K, forced by an isolated unit source at the $(I1+1)$ position in the second row from the top of subregion K. The top subregion does not require an array of this second class. (Although the above description indicates the storage sequence in regular problems, arrays RINV and RINV1 are stored as one-dimensional arrays to save storage in irregular problems due to differing storage requirements in each subregion — see below.)

Using the arrays RINV and RINV1, as determined by the JAYNOR pre-processor, which is performed in subroutine JAY, subroutine NOR calculates the solution to the partial difference equation (1) with assigned boundary conditions. This is done using a forward sweep, followed by a backward sweep, similar to the procedure described by Madala (1978), utilizing RINV and RINV1 data along the way. In the present method, the forward "particular solution sweep" and backward "homogeneous solution sweep" each requires slightly fewer

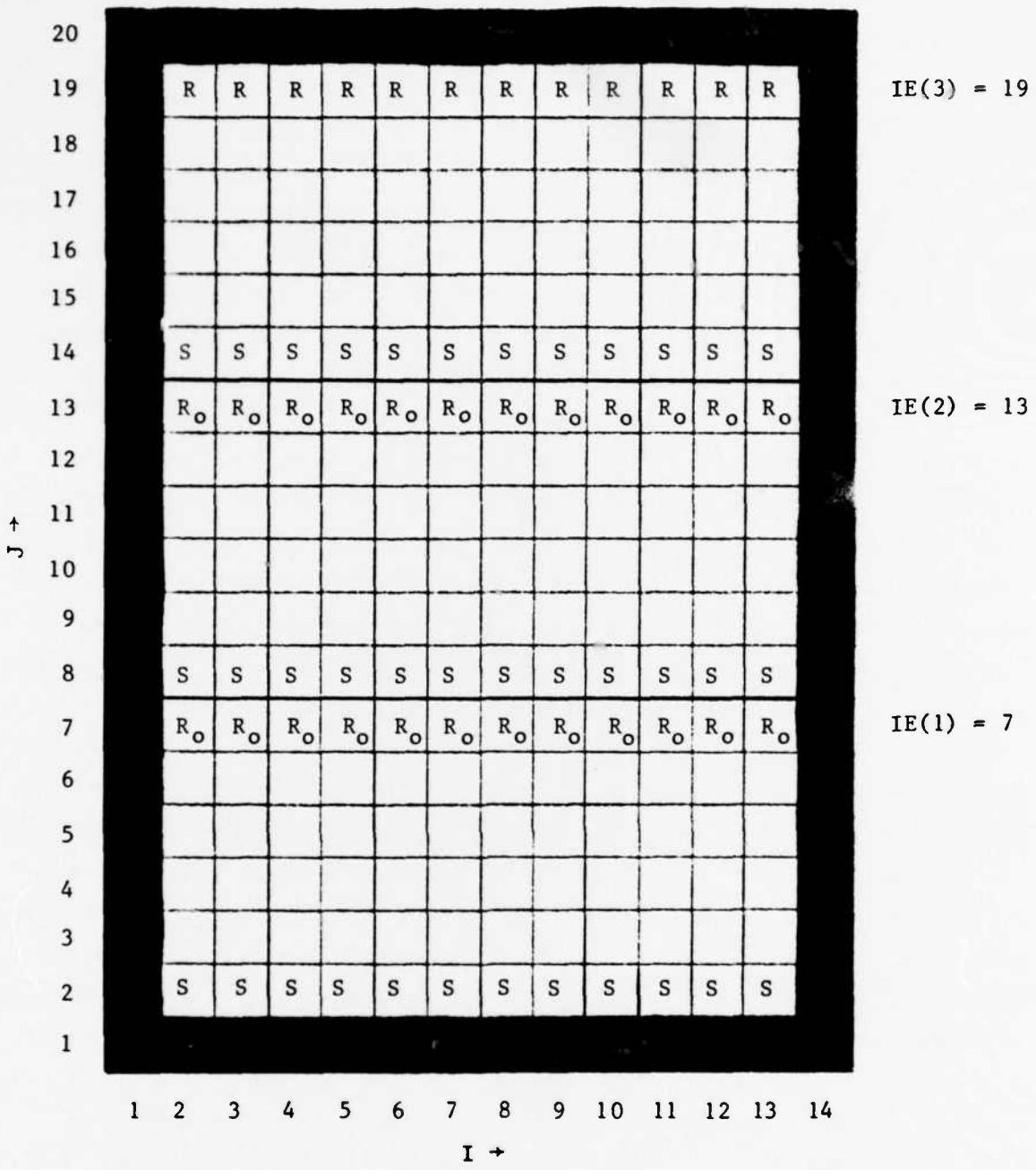


Figure 1. Application of program JAYNOR to a regular mesh. The partial difference equation is satisfied at the center of each cell, except those outside the boundary, which are shaded; the source term and solution arrays are cell-centered. Grid lines forming the cell boundaries are drawn. Grid lines between SEVP subregions are darkened. In this example, JAYNOR program topological variables are as follows: M=14, N=20, NSB=3, IE(1)=7, IE(2)=13, IE(3)=19, and the NTYPES(I,J) array has zeroes for I=1, I=14, J=1 and J=20, and ones everywhere else (there are no islands). SEVP starting vector positions are indicated by S's, and SEVP residual vector positions are indicated by R's; most of the R's have zero subscripts to indicate they are "open residual positions" (see main text).

operations than a single ordinary relaxation sweep, except for operations involving RINV and RINV1. Subroutine NOR avoids generating a homogeneous solution on the forward sweep by using RINV1 to adjust the grid point values in the second row from the top of each subregion (except the top subregion) directly during the forward sweep, instead of using RINV, RINV1, and an appropriate homogeneous sweep to create these same adjustments, as described by Madala (1978). Every other step of the present procedure is essentially the same as described by Madala when JAYNOR is applied to a regular region.

Application of JAYNOR to elliptic problems in regions with irregular boundaries and islands is similar to its application to regular problems. The differences are mainly in the definition and use of the preprocessor arrays RINV and RINV1. These differences are associated with the definition "starting vector points" and "error vector points" in each subregion. For regular problems, "starting vector points" are points in the second row from the bottom of each subregion, and "error vector points" are points in the second row from the top of each subregion (see Figure 1). When the problem is irregular, "starting vector points" are all points in the second row from the bottom of each subregion that are inside the boundary (NTYPE=1) plus all points inside the boundary that are immediately above points that are outside the boundary (NTYPE=0) (see Figure 2). Similarly, "error vector points" are all points in the second row from the top of each subregion that are inside the boundary plus all points inside the boundary that are immediately below points that are outside the boundary (see Figure 2). The general definitions for RINV and RINV1, applicable for both regular and irregular problems, are then as follows. RINV(L,M,K) is the solution at the M^{th} starting vector position in subregion K, forced by an isolated unit source at the L^{th} error vector position in subregion K, in an associated problem, as described above. RINV1(L,M,K) is the solution at the M^{th} "open" error vector position in subregion K, forced by an isolated unit source at the L^{th} error vector position in subregion K, in the associated problem, where "open" means that the point above the error vector position is inside the boundary (NTYPE=1) (see Figure 2). Since the ranges of L and M can vary from subregion to subregion in irregular problems, arrays RINV and RINV1 are treated as one-dimensional arrays to save storage.

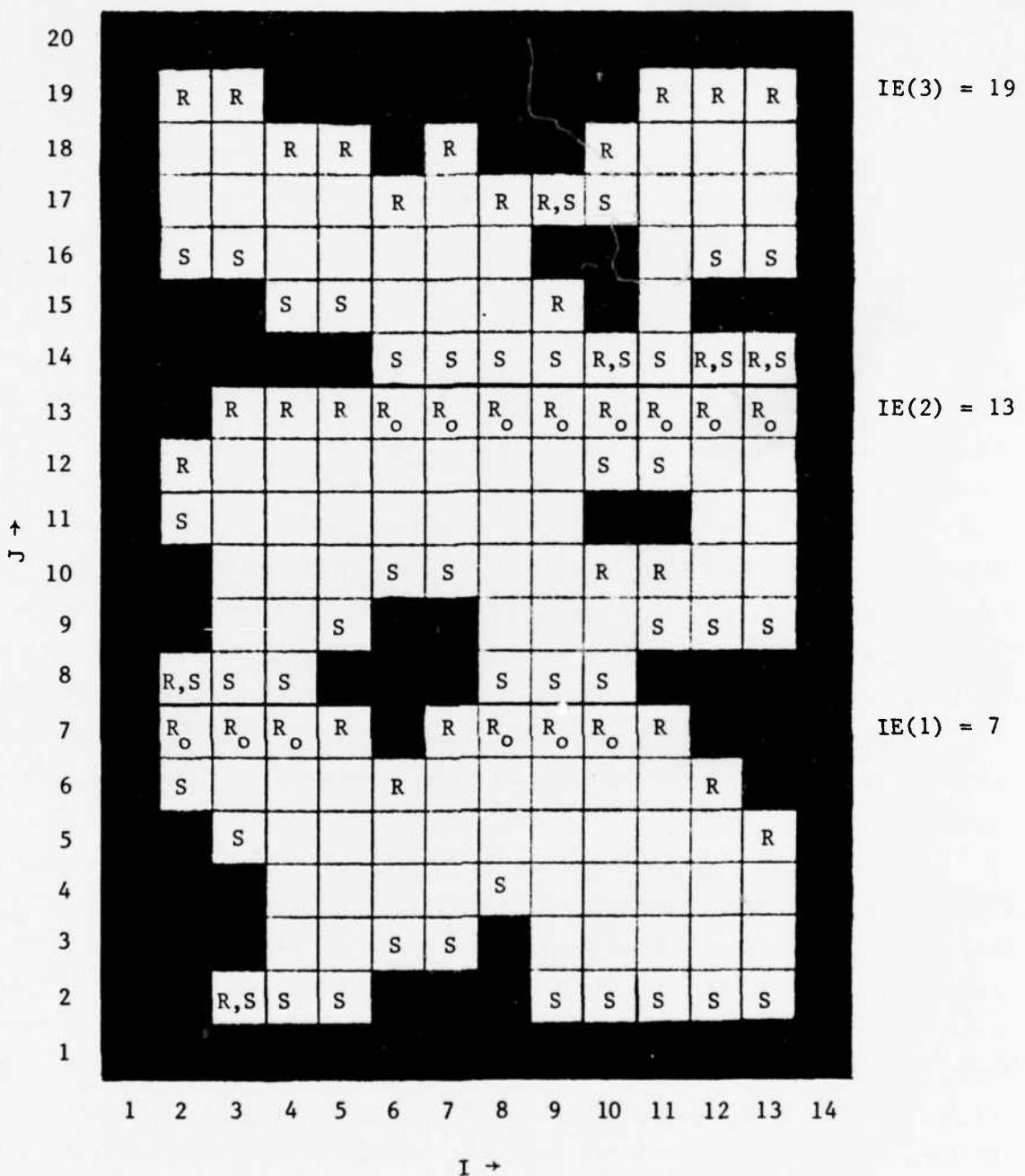


Figure 2. Application of program JAYNOR to an irregular mesh with islands. Refer to Figure 1 caption to interpret this figure. Some of the R 's have zero subscripts to indicate these are "open residual positions" (see main text). The user need not specify the R , S , and R_o positions; they are determined in subroutine TOPOL from the user-specified topological flag array NTYPE (see main text).

3. SELECTION OF MARCHING DIRECTION AND BLOCK SIZE

The choice of marching direction and block size is determined by the accuracy desired in the solution, the computing precision used, and the EVP roundoff error characteristics of the partial difference operator. (In many applications, it is desirable to use double precision on IBM computers and on other computers for which double-precision computation is about as fast as single-precision computation. This can usually be facilitated by using IMPLICIT REAL*8 type statements at the beginning of the EVP subroutines.) Detailed discussions on this are given by Dietrich (1975) and Roache (1978). In general, the choice of march direction and block size depends on a number of factors, and only rough guidelines can be given. Fortunately, this is all that is needed, since even when operating well within roundoff error restrictions, the SEVP method is robust and is usually competitive with the best of other methods, often being superior. However, it should be noted that when the geometry is regular (or nearly regular), and the partial difference operator has constant coefficients, there are transform methods (combined with a capacitance matrix to handle any irregularities) that can be superior to SEVP in terms of storage requirements and competitive in terms of operation count (Wallcraft, 1980).

A general rule-of-thumb guideline for the choice of subregion size when using SEVP is that for an elliptic partial differential equation with low to moderate diagonal dominance (as in a Poisson partial differential equation), the normalized absolute error in the application of the EVP algorithm to a given subregion is of order 5^B times the computing precision used, where B is the ratio of the subregion length in the EVP march direction to the minimum grid interval size in the other direction. For example, for a Poisson equation on a uniform mesh ($\Delta x = \Delta y$), and computing precision of 15 significant figures, the SEVP solution would be accurate to about five significant figures if the maximum subregion length were about 14 grid intervals. However, SEVP can be applied iteratively (as can any direct solver) to reduce roundoff error to any desired level above the computing

A. Wallcraft, "Capacity Matrix Techniques," Ph.D. Thesis (1980), Department of Mathematics, Imperial College, London, England S.W.7.

precision available. For example, if the roundoff error is $O(10^{-5})$ after one call to SEVP, the error can be reduced to $O(10^{-10})$ after two SEVP calls. Each successive SEVP problem is forced by the roundoff error residuals from the previous SEVP problem, and the result is added to the previous solution.

A rule for choosing the SEVP march direction is suggested by Madala (1978). Madala recommends choosing the direction for which " $A > 1$ ", where A is the ratio of the grid interval transverse to the SEVP march direction divided by the grid interval in the SEVP march direction. However, in some problems there are regions for which $A > 1$ and other regions for which $A < 1$. In such problems the use of SEVP as a direct method may be costly. A good possible alternative in such cases is to use SEVP iteratively (with variable march direction) in the same way EVP was used iteratively by Dietrich, et al. (1975); in some cases, it may be useful to use alternate forms of EVP such as described by Dietrich (1977) and by Roache (1978). Even in problems for which " $A - 1$ " has the same sign everywhere, it might not be best to march in the direction for which $A > 1$. Specifically, when $A > 1$ for a march direction with much fewer mesh points than in the other direction, the other march direction may be better even though more SEVP subregions are required, since the influence arrays RINV and RINV1 will be much smaller for a given subregion. This, of course, can be affected by the kind of computer being used (e.g., vector vs. scalar computer).

Finally, when there are islands, it is sometimes advantageous to place the SEVP subregion boundaries in such a manner that the islands straddle the boundaries, especially when this does not increase the number of SEVP subregions required; this reduces the number of starting and error vector points required and the associated auxiliary computation and storage used by the SEVP algorithm. Similarly, if there are no islands, it is desirable to place the SEVP subregion boundaries near the narrowest parts of the SEVP domain, such as the throat of an hourglass-shaped domain.

4. USING THE JAYNOR COMPUTER PROGRAM

Program JAYNOR sets up and solves a general sample problem, using preprocessor subroutines TOPOL and JAY, and repeat solution subroutine NOR; in JAYNOR program variables, the sample problem may be written:

$$\begin{aligned} & AX(I,J)*X(I,J+1) + CX(I,J)*X(I+2,J+1) + AY(I,J)*X(I+1,J) \\ & + CY(I,J)*X(I+1,J+2) + BB(I,J)*X(I+1,J+1) = F(I,J) \quad . \quad (2) \end{aligned}$$

If NTYPE(I,J) = 1, Eq. (2) is satisfied; if NTYPE(I,J) = 0, X(I,J) is left unchanged and treated as a boundary condition, where appropriate. Thus, the user must specify: the partial difference operator coefficient arrays AX, CX, AY, CY, and BB; the source term array F; the topological array NTYPE; the x-dimension of the solution array M; the y-dimension of the solution array N; the maximum (for all SEVP block subregions) number of starting vector guess positions MXGS; the number of SEVP block subregions NBLK; the dimensions of the influence coefficient arrays (RINV and RINV1) LD1 and LD2; and the grid line number at the top of each subregion, array IE. If any of the user-specified dimensions MXGS, LD1, or LD2 is too small for the specified mesh topology (NTYPE array), appropriate diagnostic messages are printed in subroutine TOPOL and the computation is halted. Most arrays have dimensions selected from the following quantities: M, M-2, N, N-2, MXGS, NBLK, and NBLK-1. Specifically, the appropriate dimensions are as follows:

(M-2,N-2): AX, AY, CX, CY, BB, Q, F
(M,N): RECUR, X, H, ERR, XX, NTYPE
Also: DUM0(MXGS, NBLK-1), DUM1(MXGS), DUM2(MXGS),
ILFT(N), IRGT(N), IGES(MXGS, NBLK), JGES(MXGS, NBLK),
ICHK(MXGS, NBLK), JCHK(MXGS, NBLK), NEV(NBLK),
LINK(M-2, NBLK-1), NLINK(NBLK-1), IE(NBLK), L1(NBLK),
and L2(NBLK-1).

The maximum lengths required by RINV and RINV1 are:

RINV(NBLK*MXGS*MXGS), RINV1((NBLK-1)*MXGS*(M-2)).

A complete alphabetical list of the JAYNOR program significant variables and their definitions follows. User-specified variables are denoted by an asterisk.

TABLE 1: JAYNOR VARIABLES AND DEFINITIONS

Variable Name	Variable Definition
AX(I,J)*	Left coefficient of difference operator at mesh point (I+1,J+1)
AY(I,J)*	Bottom coefficient of difference operator at mesh point (I+1,J+1)
BB(I,J)*	Central coefficient of difference operator at mesh point (I+1,J+1)
CX(I,J)*	Right coefficient of difference operator at mesh point (I+1,J+1)
CY(I,J)*	Top coefficient of difference operator at mesh point (I+1,J+1)
ERR(I,J)	Normalized residual array
ERSUM	Mean absolute residual
F(I,J)*	Source term for difference operator at mesh point (I+1,J+1)
FSUM	Mean absolute source term
H(I,J)	Almost-homogeneous solution component array calculated on backward SEVP sweep
(IC,JC)	Mid-point of island location in SEVP sample problem
ICHK(N,NB)	The N th residual position in the NB th block is on the ICHK(N,NB) vertical mesh line

(continued)

TABLE 1 (continued)

Variable Name	Variable Definition
IE(NB)	The top azimuthal mesh line inside the NB th block (IE(NBLK)=N-1)
IGES(N,NB)	The N th starting vector position in the NB th block is on the IGES(N,NB) vertical mesh line
ILFT(J)	The left-most point inside the boundary on horizontal mesh line J
IRGT(J)	The right-most point inside the boundary on horizontal mesh line J
JCHK(N,NB)	The N th residual position in the NB th block is on the JCHK(N,NB) horizontal mesh line
JGES(N,NB)	The N th starting vector position in the NB th block is on the JGES(N,NB) horizontal mesh line
LD1*	The dimension of array RINV
LD2*	The dimension of array RINV1
LINK(N,NB)	The N th open residual position in the NB th block is at the LINK(N,NB) residual position (I=ICHK(LINK(N,NB),NB), J=JCHK(LINK(N,NB),NB)=IE(NB))
L1(NB)	The total number of elements in array RINV for blocks below block NB
L2(NB)	The total number of elements in array RINV1 for blocks below block NB
M*	The number of vertical mesh lines spanning the problem to be solved
MXGS*	The maximum number of guess points to be encountered over all blocks (can be larger)
N*	The number of horizontal mesh lines spanning the problem to be solved
NBLK*	The number of blocks (SEVP subregions) to be used
NEV(NB)	The number of starting vector (or residual) positions in block NB

(continued)

TABLE 1 (continued)

Variable Name	Variable Definition
NLINK(NB)	The number of open residual positions in block NB
NTYPE(I,J)*	Topological flag array (NTYPE(I,J)=1 if the partial difference operator is to be satisfied at the grid point located at the intersection of the I th vertical mesh line and J th horizontal mesh line; NTYPE(I,J)=0 otherwise)
Q(I,J)	Same as F(I,J)
RECUR(I,J)	Real array of 1's and 0's related to NTYPE
RINV	Influence coefficient array relating unit source terms at error vector positions to solutions at starting vector positions
RINV1	Influence coefficient array relating unit source terms at error vector positions to solutions at error vector positions
X(I,J)	Solution at mesh point (I,J)
XX(I,J)	Same as X(I,J)

*User-specified variable

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APPENDIX I

TRANSFORMATION TO DIRICHLET BOUNDARY CONDITIONS FROM NEUMAN OR MIXED BOUNDARY CONDITIONS

Using the notation of Eq. (1) in the main text, suppose the point ($i=I+1, j=J+1$) is inside the boundary, whereby Eq. (1) applies with $i=I$ and $j=J$, and suppose that the point ($i=I, j=J+1$) lies outside the boundary, whereby Eq. (1) does not apply with $i=I-1$ and $j=J$. Instead, we apply a "boundary condition" of the form

$$X_{I,J+1} = \beta - \alpha \cdot X_{I+1,J+1}, \quad (3)$$

where α and β are specified. If $\alpha = 0$ we have a Dirichlet condition, and no adjustment is necessary before using program JAYNOR. However, if $\alpha \neq 0$ we can use Eq. (3) to eliminate $X_{I,J+1}$ from the equation set. We do this by substituting Eq. (3) into Eq. (1) with $i=I$ and $j=J$ to obtain

$$\begin{aligned} A'_{I,J} \cdot X_{I,J+1} + B'_{I,J} \cdot X_{I+2,J+1} + C'_{I,J} \cdot X_{I+1,J} \\ + D'_{I,J} \cdot X_{I+1,J+2} + E'_{I,J} \cdot X_{I+1,J+1} = F'_{I,J} \end{aligned}, \quad (4)$$

where $A'_{I,J} = 0$, $E'_{I,J} = E_{I,J} - A_{I,J} \cdot \alpha$, and $F'_{I,J} = F_{I,J} - A_{I,J} \cdot \beta$.

The modified partial difference equation (4) can be solved by program JAYNOR; the value of $X_{I,J+1}$ does not matter in this solution procedure, although the user is free to calculate the appropriate value, after JAYNOR gives the correct $X_{I+1,J+1}$ value, using Eq. (3). Mixed boundary conditions in the other three directions from the point ($I+1, J+1$) can be treated in an analogous manner.

APPENDIX II

COMPUTER LISTINGS OF FOUR SEVP VERSIONS

The main text describes the most general version of program JAYNOR in detail. This is Version 2. The differences between each successive version and the previous one are indicated by comment cards in the main program JAYNOR.

The listings for four versions follow.

VERSION 2

```

1.      01      PROGRAM JAYNOR(INPUT,OUTPUT,TAPE5=TPUT,TAPE6=OUTPUT)
2.      00      COMMON/SEVP/AX(7,9),AY(7,9),BB(7,9),CX(7,9),CY(7,9),Q(7,9),F(7,9),
3.      00      A RECUR(9,I1),RINV(SBB),RINV1(308),DUMC(I4,2),DUM1(14),
4.      00      B DUM2(I4),X(9,11),M(9,11),ERR(9,I1),XX(9,I1),ILFT(I1),
5.      00      C IRGT(I1),IGES(I4,3),JGES(I4,3),TCHK(I4,3),JCHK(I4,3),NEV(3),
6.      00      D LTNK(I7,2),NLINK(2),NTYPE(9,11),TE(3),LI(3),L2(2)
7.      00      DATA M,N,MXGS,NBLK,L0I,L02/9,I1,14,3,588,308/
8.      00      DATA TE/4,7,10/
9.      00
10.     00      C DIETRICH'S MODIFIED MADALA SOLVER, VERSION 2.
11.     00      C THIS VERSION IS IDENTICAL TO VERSION 1, EXCEPT
12.     00      C FOR MAKING RINV AND RINV1 INTO 1-DIMENSIONAL ARRAYS, AND MODIFYING
13.     00      C BOOKKEEPING TO REDUCE STORAGE REQUIREMENTS.
14.     00      C EQUATION TO BE SOLVED IS
15.     00      C NTYPE(I+I,J+I) * (AX(I,J) + X(I,J+I) + CX(I,J) + X(I+2,J+1)
16.     00      C           + AY(I,J) + X(I+1,J) + CY(I,J) + X(I+1,J+2))
17.     00      C           + (2 - NTYPE(I+I,J+I)) * BB(I,J) + X(I+1,J+1)
18.     00      C           + NTYPE(I+1,J+I) * F(I,J), ((I=0,I,...,M-1),(J=0,I,...,N-1))
19.     00      C WHERE TERMS OUTSIDE DIMENSION BOUNDS ARE INTERPRETED AS ZERO'S.
20.     00      C THUS, IF NTYPE(I,J)=0, X(I,J) IS LEFT UNCHANGED.
21.     00      MI=M-1
22.     00      NI=N-1
23.     00      M2=M-2
24.     00      N2=N-2
25.     00      NBLK1=NBLK-I
26.     00      DO 80 J=1,N
27.     00      DO 80 I=1,M
28.     00      8C NTYPE(I,J)=0.
29.     00      DO 90 J=2,NI
30.     00      DO 90 I=2,MI
31.     00      9C NTYPE(I,J)=I
32.     00      DO 2000 JC=2,NI
33.     00      DO 2000 IC=2,M1
34.     00      NTYPE(IC,JC)=0
35.     00      NTYPE(IC-I,JC)=0
36.     00      NTYPE(IC+I,JC)=C
37.     00      NTYPE(IC,JC-1)=0
38.     00      NTYPE(IC,JC+1)=0
39.     00      WRITE(6,92) ((NTYPE(I,J),I=I,M),J=I,N)
40.     00      92 FORMAT(10X,9I3)
41.     01      CALL TOPOL(RECUR,ILFT,IRGT,IGES,JGES,ICHK,JCHK,NEV,LINK,NLINK,
42.     00      A NTYPE,IE,M,N,MI,N1,M2,NBLK,NBLK1,MXGS,L1,L2,L0I,L02)
43.     00      DO 100 J=I,N2
44.     00      DO 100 I=I,M2
45.     00      AX(I,J)=1.-.0I*(T-4)**2
46.     00      CX(I,J)=I.-.0I*(I-4)**2
47.     00      AY(I,J)=I.-.0I*(J-5)**2
48.     00      CY(I,J)=1.-.0I*(J-5)**2
49.     00      BB(I,J)=AX(I,J)-AY(I,J)-CX(I,J)-CY(I,J)-.1
50.     00      100 CONTINUE
51.     00      DENOM=(.25*M*N)**2
52.     00      DO 170 J=I,N
53.     00      DO 170 I=1,M
54.     00      X(I,J)=(I-1)*(I-M)+(J-I)*(J-N)/DENOM
55.     00      XX(I,J)=X(I,J)*(1.-NTYPE(I,J))
56.     00      170 CONTINUE
57.     00      FSUM=0.

```

VERSION 2
(continued)

```

58.      00      00  I71 J=2,N1
59.      00      00  L=J-1
60.      00      00  00  I71 I=2,M1
61.      00      00  K=I-I
62.      00      00  FIK,L)=AXIK,L)*X(K,J)+AYIK,L)*X(I,I,L)+CXIK,L)*X(I+1,J)+CYIK,L)*
63.      00      00  A X(I,J+1)+BBIK,L)*X(I,J)
64.      00      00  171 FSUM=FSUM+ABS(FIK,L))
65.      00      00  FSUM=M2*N2/FSUM
66.      00      00  T0=SECONDTIM
67.      01      01  CALL JAYJAX,AY,BB,CX,CY,RINV,RINV1,M,RECUR,ILFT,IRGT,IGES,
68.      00      00  AICHK,JCHK,NEV,LINK,NLINK,NTYPE,IE,M,N,M2,N2,NBLK,NBLK1,MX65,L1,L2)
69.      00      00  TIMSUM=0.
70.      00      00  DO 181 J=2,N1
71.      00      00  DO 181 I=2,M1
72.      00      00  OI1-I,J-I)=FI1-I,J-I)
73.      00      00  181 X(I,J)=0.
74.      00      00  ALINE=0.
75.      00      00  T0=SECONDTIM
76.      01      01  CALL NORIAJ,AY,BB,CX,CY,RINV,RINV1,DUMD,DUM1,DUM2,O,M,XX,RECUR,
77.      00      00  A ILFT,IRGT,IGES,IGES,ICHK,JCHK,NEV,LINK,NLINK,IE,M,N,M2,N2,NBLK,
78.      00      00  B NBLK1,MXGS,L1,L2)
79.      00      00  DO 990 J=2,N1
80.      00      00  DO 990 I=2,M1
81.      00      00  950 X(I,J)=X(I,J)+XX(I,J)
82.      00      00  DO 991 J=I,N
83.      00      00  DO 991 I=1,M
84.      00      00  951 XX(I,J)=0.
85.      00      00  DO 992 J=2,N1
86.      00      00  L=J-1
87.      00      00  DO 992 I=2,M1
88.      00      00  K=I-1
89.      00      00  992 OIK,L)=FIK,L)-AYIK,L)*X(I,J-I)-AXIK,L)*X(I-1,J)-BBIK,L)*
90.      00      00  A CXIK,L)*X(I+1,J)-CYIK,L)*X(I,J+1)
91.      00      00  1C00 CONTINUE
92.      00      00  9C5 FORMAT(1X,1P15E8.1)
93.      00      00  ERSUM=0.
94.      00      00  DO 740 I=2,M1
95.      00      00  K=I-1
96.      00      00  DO 740 J=2,N1
97.      00      00  L=J-1
98.      00      00  ERRII,J)=FIK,L)-AYIK,L)*X(I,J-I)-AXIK,L)*X(I-1,J)-BBIK,L)*
99.      00      00  A X(I,J)-CXIK,L)*X(I-1,J)-CYIK,L)*X(I,J+1)
100.     00      00  ERRII,J)=ERRII,J)+NTYPEII,J)
101.     00      00  ERSUM=ERSUM+ABS1ERRII,J)
102.     00      00  740 ERRII,J)=ERRII,J)+FSUM
103.     00      00  ERSUM=ERSUM+FSUM/(M2*N2)
104.     00      00  WR1TE16,904)
105.     00      00  9C4 FORMAT(1/40X,2GHNORMALIZED RESIDUALS)
106.     00      00  DO 742 I=2,M1
107.     00      00  WR1TE16,905)(ERRII,J),J=2,N1)
108.     00      00  742 CONTINUE
109.     00      00  WR1TE16,906) ERSUM
110.     00      00  9C6 FORMAT(1/37H MEAN ABSOLUTE NORMALIZED RESIDUAL = ,1PE9.2)
111.     00      00  IF 1ERSUM.GT.1.E-4) STOP
112.     00      00  IF 1TC.NE.2) NTYPE(1C-I,JC)=I
113.     00      00  IF 1TC.NE.M1) NTYPE(1C+I,JC)=I
114.     00      00  IF 1JC.NE.?) NTYPE(1C,JC-I)=I
115.     00      00  IF 1JC.NE.N1) NTYPE(1C,JC+I)=I
116.     00      00  2C00 NTYPE(1C,JC)=I
117.     00      00  ENO

```

VERSION 2
(continued)

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1.      02      SUBROUTINE JAYIA(X,AY,BB,CX,CY,RINV1,H,RECUR,ILFT,
2.      00      IRG1T,IGES,JGES,ICHK,JCHK,NEV,LINK,NLINK,NTYPE,IE,MD,ND,M2,N2,
3.      00      A NBLK,NBLKI,MXGS,L1,L2)
4.      00      DIMENSION AX(M2,N2),AY(M2,N2),BB(M2,N2),CX(M2,N2),CY(M2,N2),
5.      00      A RINV1(),RINV1(),RECUR(MD,ND),H(MD,ND),
6.      00      B ILFT(ND),IRGT(ND),L1(NBLK),L2(NBLK),IGES(MXGS,NBLK),
7.      00      C JGES(MXGS,NBLK),ICHK(MXGS,NBLK),JCHK(MXGS,NBLK),NEV(NBLK),
8.      00      D LINK(M2,NBLK),NLINK(NBLK),NTYPE(MD,ND),IE(NBLK)
9.      00      JL=I
10.     00      LD=L1(NBLK)
11.     00      NB=G
12.     00      100  NB=NB+1
13.     00      NGES=NEV(NB)
14.     00      NGES1=NEV(NB-1)
15.     00      LA=L1(NB)
16.     00      LB=L2(NB-1)
17.     00      LC=L2(NB)
18.     00      JM=IE(NB)
19.     00      JHM=JM+1
20.     00      JMM=JM-2
21.     00      00 250  NG=1,NGES
22.     00      IG=IGE5(NG,NB)
23.     00      JG=JGES(NG,NB)
24.     00      JF=JG-1
25.     00      00 210  J=JL,JHM
26.     00      00 210  I=I,MD
27.     00      210  H(I,J)=0.
28.     00      H(IG,JG)=I.
29.     00      IF (JF.GT.JMM) GO TO 228
30.     00      IF (NTYPE(IG,JF).EQ.0) GO TO 220
31.     00      NOP=NLINK(NB-1)
32.     00      00 214  N=1,NOP
33.     00      N=NLINK(N,NB-1)
34.     00      IF (ICHK(H,NB-1).EQ.16.AND.JCHK(H,NB-1).EQ.JF) GO TO 215
35.     00      214  CONTINUE
36.     00      215  00 218  N=I,NOP
37.     00      L=NLINK(N,NB-1)
38.     00      218  H(ICHK(L,NB-1),JF)=RINV1(LB+(N-1)*NGES1+H)*CY(IG-1,JG-2)
39.     00      00 225  J=JF,JHM
40.     00      IL=ILFT(J+2)-1
41.     00      IR=IRGT(J+2)-1
42.     00      00 225  I=IL,IR
43.     00      225  H(I+1,J+2)=-RECUR(I+1,J+2)*(AX(I,J)*H(I,J+1)+AY(I,J)*H(I+1,J)+
44.     00      A BB(I,J)*H(I+1,J+1)*CX(I,J)*H(I+2,J+1))/CY(I,J)
45.     00      00 230  NC=1,NGES
46.     00      I=1,ICHK(1,NC,NB)-1
47.     00      J=JCHM(1,NC,NB)-1
48.     00      230  RINV1(LA+(NC-1)*NGES+NC)=AX(1,J)*H(I,J+1)+AY(1,J)*H(I+1,J)+BB(I,J)*
49.     00      A H(I+1,J+1)*CX(I,J)*H(I+2,J+1)
50.     00      IF (NB.EQ.NBLK) GO TO 250
51.     00      NOP=NLINK(NB)
52.     00      J=IE(NB)
53.     00      00 240  N=1,NOP
54.     00      N=NLINK(N,NB)
55.     00      240  RINV1(LD+(N-1)*NGES+NG)=H(ICHK(H,NB),J)
56.     00      250  H(IG,JG)=0.
57.     00      CALL MATINV(RINV1(LA+1),NGES,NGES)
58.     00      IF (NB.EQ.NBLK) RETURN
59.     00      00 260  I=I,NGES
60.     00      00 260  J=I,NOP
61.     00      RINV1(LC+(J-1)*NGES+I)=0.
62.     00      00 260  K=I,NGES
63.     00      260  RINV1(LC+(J-1)*NGES+I)=RINV1(LC+(J-1)*NGES+I)-
64.     00      A RINV1(LA+(K-1)*NGES+I)*RINV1(LD+(J-1)*NGES+K)
65.     00      C RINV1(I,J,ND) IS THE "ALMOST HOMOGENEOUS" SOLUTION AT THE J-TH OPEN
66.     00      C RESIDUAL POSITION FORCED BY A RESIDUAL VALUE OF 1 AT THE I-TH
67.     00      C RESIDUAL POSITION. HOMOGENEOUS B.C.'S ARE ASSUMED EVERYWHERE,
68.     00      C INCLUDING THE TOP OF THE PRESENT SUBREGION NB.
69.     00      JL=JM
70.     00      00 TO 100
71.     00      ENO

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VERSION 2
(continued)

```

1.      01      SUBROUTINE NOR(AX,AY,BB,CX,CY,RINV,RINV1,DUMD,DUM1,DUM2,F,M,X,
2.      00      IRECUR,ILFT,IRGT,IGES,ICHK,JCHK,NEV,LINK,NLINK,IE,MD,ND,M2,
3.      00      A N2,NBLK,NBLK1,MXGS,L1,L2)
4.      00      DIMENSION AX(M2,N2),AY(M2,N2),BB(M2,N2),CX(M2,N2),CY(M2,N2),
5.      00      A RINV1),RINV1(1),RECUR(MD,ND),H(MD,ND),
6.      00      B ILFT(ND),IRGT(ND),L1(NBLK1),L2(NBLK1),GES(MXGS,NBLK),
7.      00      C JGES(MXGS,NBLK1),JCHK(MXGS,NBLK1),JCHK(MXGS,NBLK1),NEV(NBLK),
8.      00      D LINK1(M2,NBLK1),NLINK(NBLK1),IE(NBLK1)
9.      00      DIMENSION DUMD(M2,NBLK1),DUM1(MXGS),DUM2(MXGS),F(M2,N2),X(MD,ND)
10.     00      COMMON/BSM/AIT,ALINE,BIT,BLINE,ITHMAX
11.     00      JS=1
12.     00      DO 150 NB=1,NBLK
13.     00      LC=L2(NB)
14.     00      JF=IE(NB)-2
15.     00      DO 105 J=JS,JF
16.     00      IL=ILFT(J+2)-1
17.     00      IR=IRGT(J+2)-1
18.     00      DO 105 I=IL,IR
19.     00      1C5 X(I+1,J+2)=RECUR(I+1,J+2)*(F(I,J)-AX(I,J)*X(I,J+1)-AY(I,J)*
20.     00      A X(I+1,J)-BB(I,J)*X(I+1,J+1)-CX(I,J)*X(I+2,J+1))/CY(I,J)*(1.-
21.     00      B RECUR(I+1,J+2))*X(I+1,J+2)
22.     00      IF (NB.EQ.NBLK) GO TO 150
23.     00      NGES=NEV(NB)
24.     00      DO 115 N=1,NGES
25.     00      I=ICHK(N,NB)-1
26.     00      J=JCHK(N,NB)-1
27.     00      115 DUM1(N)=F(I,J)-AX(I,J)*X(I,J+1)-AY(I,J)*X(I+1,J)-BB(I,J)*
28.     00      A X(I+1,J+1)-CX(I,J)*X(I+2,J+1)-CY(I,J)*X(I+1,J+2)
29.     00      NOP=NLINK(NB)
30.     00      J=IE(NB)
31.     00      DO 120 N=1,NOP
32.     00      DUM2(N)=D.
33.     00      DO 118 M=1,NGES
34.     00      118 DUM2(N)=DUM2(N)+DUM1(N)*RINV1(LC+(N-1)*NGES+N)
35.     00      M=LINK(N,NB)
36.     00      I=ICHK(M,NB)
37.     00      DUM0(M,NB)=X(I,J)
38.     00      120 X(I,J)=X(I,J)-DUM2(N)
39.     00      150 JS=IE(NB)
40.     00      DO 300 NB1=1,NBLK
41.     00      NB=NBLK-NB1+1
42.     00      JS=1
43.     00      IF (NB.NE.1) JS=IE(NB-1)
44.     00      JF=IE(NB)-2
45.     00      LA=L1(NB)
46.     00      LB=L2(NB-1)
47.     00      NGES=NEV(NB)
48.     00      IF (NB.EQ.NBLK) GO TO 201
49.     00      J=IE(NB)
50.     00      NOP=NLINK(NB)
51.     00      DO 200 N=1,NOP
52.     00      M=LINK(N,NB)
53.     00      I=ICHK(M,NB)
54.     00      200 X(I,J)=DUMC(N,NB)
55.     00      201 CONTINUE
56.     00      N=IE(NB)
57.     00      DO 202 JS=N,N

```

VERSION 2
(continued)

```

58.      00      00 202 I=1,N0
59.      00      202 H(I,J)=0.
60.      00      00 210 N=1,NGES
61.      00      I=JCHK(N,NB)-1
62.      00      J=JCHK(N,NB)-1
63.      00      210 DUM1(N)=F(I,J)*X(I,J+1)-AY(I,J)*X(I+1,J)-BB(I,J)*
64.      00      A X(I+1,J+1)-CX(I,J)*X(I+2,J+1)-CY(I,J)*X(I+1,J+2)
65.      00      00 220 N=1,NGES
66.      00      DUM2(N)=0.
67.      00      00 218 DUM2(N)=DUM2(N)+DUM1(M)*RINV(LA*(N-1)*NGES+M)
68.      00      I=JGES(N,NB)
69.      00      J=JGFS(N,NB)
70.      00      H(I,J)=DUM2(N)
71.      00      220 X(I,J)=X(I,J)+DUM2(N)
72.      00      IF (NB.EQ.1) GO TO 250
73.      00      00 222 N=1,MXGS
74.      00      DUM1(N)=0.
75.      00      NOP=NLINK(NB-1)
76.      00      J=JS
77.      00      00 230 N=1,NOP
78.      00      M=LINK(N,NB-1)
79.      00      I=JCHK(M,NB-1)
80.      00      230 DUM1(M)=H(I,J+1)*CY(I-1,J-I)
81.      00      C WHEN BOUNDARY IS IRREGULAR, THERE IS SOME WASTED CALCULATION HERE IN
82.      00      C ORDER TO AVOID LOGICAL DECISIONS OR EXTRA STORAGE NGES COULD BE
83.      00      C REPLACED BY NLINK(NB))
84.      00      NGES=NEV(NB-1)
85.      00      00 240 N=1,NOP
86.      00      DUM2(N)=0.
87.      00      00 238 M=1,NGES
88.      00      238 DUM2(N)=DUM2(N)+DUM1(M)*RINV(LB*(N-1)*NGES+M)
89.      00      M=LINK(N,NB-1)
90.      00      240 H(ICHK(M,NB-1),JCHK(M,NB-1))=DUM2(N)
91.      00      250 00 300 JS, JF
92.      00      IL=ILFT(I+2)-1
93.      00      IR=IRGT(I+2)-1
94.      00      00 300 I=IL,IR
95.      00      DUM1(I)=RECUR(I+1,J+2)*(AX(I,J)*H(I,J+1)+AY(I,J)*H(I+1,J)*
96.      00      A BB(I,J)*H(I+1,J+1)+CX(I,J)*H(I+2,J+1))/CY(I,J)
97.      00      H(I+1,J+2)=DUM1(I)+(I.-RECUR(I+1,J+2))*H(I+1,J+2)
98.      00      300 X(I+1,J+2)=X(I+1,J+2)+DUM1(I)
99.      00      ENO
00.      00

```

VERSION 2
(continued)

```

1.      CI      SUBROUTINE TOPOL(IRECUR,ILFT,IRGT,IGES,JGES,ICHK,JCHK,NEV,LINK,
2.      00      1NLINK,NTYPE,IE,M,N,M1,M2,NBLK,NBLK1,MXGS,L1,L2,LD1,LD2)
3.      00      DIMENSION RECUR(M,N), ILFT(N),IRGT(N),IGES(MXGS,NBLK),
4.      00      A(JGES(MXGS,NBLK)),ICHK(MXGS,NBLK),JCHK(MXGS,NBLK),NEV(NBLK),
5.      00      BLINK(M2,NBLK1),NLINK(NBLK1),NTYPE(M,N),IE(NBLK),L1(NBLK),L2(NBLK)
6.      00      DO 10 J=2,N1
7.      00      I=0
8.      00      1=I+1
9.      00      IF (NTYPE(I,J).EQ.NTYPE(I,J-1)).EQ.0.AND.I.LT.M1) GO TO 2
10.     00      ILFT(I,J)=I
11.     00      I=M
12.     00      I=I-1
13.     00      IF (NTYPE(I,J).EQ.NTYPE(I,J-1)).EQ.0.AND.I.GT.I) GO TO 4
14.     00      IRGT(I,J)=I
15.     00      IRGT(1)=2
16.     00      ILFT(1)=2
17.     00      ILFT(N)=2
18.     00      IRGT(N)=2
19.     00      DO 15 J=2,N1
20.     00      IF ((ILFT(I,J).LE.IRG(T)(J)) GO TO 15
21.     00      IRGT(J)=2
22.     00      ILFT(J)=2
23.     00      15 CONTINUE
24.     00      JMIN=2
25.     00      MXLNK=0
26.     00      L1(1)=0
27.     00      L2(1)=0
28.     00      DO 100 NB=1,NBLK
29.     00      NGES=0
30.     00      NCCHK=0
31.     00      JMAX=IE(NB)
32.     00      DO 40 I=2,M1
33.     00      NM=NTYPE(I,JMIN-1)
34.     00      NU=NTYPE(I,JMIN)
35.     00      DO 40 J=JMIN,JMAX
36.     00      NL=NM
37.     00      NM=NU
38.     00      RECUR(I,J)=NM
39.     00      IF (NL.EQ.0) RECUR(I,J)=0.
40.     00      NU=NTYPE(I,J+1)
41.     00      IF (NM.EQ.0) GO TO 40
42.     00      IF (NL.EQ.1.AND.J.NE.JMIN) GO TO 20
43.     00      NGES=NGES+1
44.     00      IF (NGES.GT.MXGS) GO TO 200
45.     00      IGES(NGES,NB)=I
46.     00      JGES(NGES,NB)=J
47.     00      20 IF (NU.EQ.1.AND.J.NE.JMAX) GO TO 40
48.     00      NCCHK=NCCHK+1
49.     00      ICHK(NCCHK,NB)=I
50.     00      JCHK(NCCHK,NB)=J
51.     00      40 CONTINUE
52.     00      NEV(NB)=NGES
53.     00      IF (NB.EQ.NBLK) GO TO 100
54.     00      NLNK=0
55.     00      DO 80 K=1,NGES
56.     00      I=ICHK(K,NB)
57.     00      J=JCHK(K,NB)
58.     00      IF (NTYPE(I,J+1).EQ.0) GO TO 80
59.     00      NLNK=NLNK+1
60.     00      LINK(NLNK,NB)=K
61.     00      80 CONTINUE
62.     00      NLINK(NB)=NLNK
63.     00      MXLNK=MAXJ(MXLNK,NLNK+NGES)
64.     00      L1(NB+1)=L1(NB)+NGES+2
65.     00      IF (NB.NE.NBLK) L2(NB+1)=L2(NB)+NGES+NLNK
66.     00      LRINV1=L2(NB)+NGES+NLNK
67.     00      100 JMIN=JMAX+1
68.     00      C IF MXLNK.GT.NGES, THERE IS SOME WASTED STORAGE THAT IS VERY
69.     00      C DIFFICULT TO PROGRAM AROUND, BECAUSE OF THE DUAL ROLE OF THE LAST
70.     00      C BLOCK OF RINV, WHICH IS USED AS A SCRATCH AREA FOR RINV1 DATA.
71.     00      C LRINV=L1(NBLK)+MAXJ(MXLNK,NGES+2)
72.     00      C WRITE(6,150) LRINV,LRINV1
73.     00      150 FORMAT(SSH Dimensions Required for RINV and RINV1 are LD1, LD2 = ,
74.     00      A15,2X,I5,1H./124H IF EITHER INPUT VALUE LD1 OR LD2 IS TOO SMALL, THE
75.     00      C COMPUTATION IS HALTED HERE, AND THE USER MUST INCREASE LD1 AND/
76.     00      C LD2 TO /22M THE INDICATED VALUES.)
77.     00      C IF ((LD1.LT.LRINV).OR.(LD2.LT.LRINV1)) STOP
78.     00      RETURN
79.     00      200 WRITE(6,202)
80.     00      202 FORMAT(92H Dimension parameter MXGS IS TOO SMALL FOR SPECIFIED TOP
81.     00      AGRAPHY AND SUBREGION SPECIFICATION.)
82.     00      STOP
83.     00      END

```

VERSION 2
(continued)

```

1.      00      SUBROUTINE MATINV(B,N,M)
2.      00      DIMENSION B(N,1),B1(100),B2(100)
3.      00      M1=M-1
4.      00      DO 110 I=1,M1
5.      00      B1(I)=1./B(I,I)
6.      00      B(I,I)=1.0
7.      00      DO 112 J=1,M
8.      00      112 B(I,J)=B(I,J)*B1(I)
9.      00      IP1=I+1
10.     00      DO 120 I1=IP1,M
11.     00      120 B1(I1)=B(I1,I)
12.     00      DO 125 I1=IP1,M
13.     00      125 B(I1,I)=0.
14.     00      DO 127 J=1,M
15.     00      127 B2(J)=B(I,J)
16.     00      DO 135 I1=IP1,M
17.     00      DO 135 J=1,M
18.     00      135 B(I1,J)=B(I1,J)-B1(I1)*B2(J)
19.     00      130 CONTINUE
20.     00      B1(I)=1./B(M,M)
21.     00      B(M,M)=1.
22.     00      DO 140 J=1,M
23.     00      140 B(M,J)=B(M,J)*B1(I)
24.     00      DO 150 I=2,M
25.     00      DO 155 I2=1,I
26.     00      155 B1(I2)=B(I2,I)
27.     00      IM1=I-1
28.     00      DO 156 I2=1,IM1
29.     00      156 B(I2,I)=0.
30.     00      DO 157 J=1,M
31.     00      157 B2(J)=B(I,J)
32.     00      IM1=I-1
33.     00      DO 160 I2=1,IM1
34.     00      DO 160 J=1,M
35.     00      160 B(I2,J)=B(I2,J)-B1(I2)*B2(J)
36.     00      150 CONTINUE
37.     00      END

```

VERSION 3

```

1.      01      PROGRAM JAYNOR(INPUT,OUTPUT,TAPES=INPUT,TAPE6=OUTPUT)
2.      00      COMMON/SEVP/AX(7,9),AY(7,9),BB(7,9),CX(7,9),Q17,9),F17,9),
3.      00      A RECUR(9,II),RINV(588),RINV1(308),DUMD(14,2),DUMI(14),
4.      00      B DUM2(14),X19,II),M19,II),ERR19,II),XX19,II),ILFT(11),
5.      00      C IRGT(II),IGES(14,3),JGES14,3),ICHK(14,3),JCHK(14,3),NEV(3),
6.      00      D LINK17,2),NLINK(2),NTYPE19,II),IE(3),LT(3),L2(2)
7.      00      DATA MN,MXGS,NBLK,L0J,L02/9,II,14,3,588,308/
8.      00      DATA IE/4,7,10/
9.      00
10.     00      C OETRICH'S MODIFIED MADALA SOLVER, VERSION 3.
11.     00      C THIS VERSION IS IDENTICAL TO VERSION 2, EXCEPT
12.     00      C CY HAS BEEN ELIMINATED. (BY DIVIDING THROUGH THE ELLIPTIC EQUATION
13.     00      C BY THE COEFFICIENT OF X(I+1,J+2), THE USER CAN USUALLY GET THIS FORM
14.     00      C IN A STRAIGHTFORWARD MANNER. IF THIS COEFFICIENT VANISHES NEAR UPPER
15.     00      C J-BOUNDARIES, A MODIFIED FORM OF THIS VERSION, IN WHICH THE RESIDUAL
16.     00      C CALCULATION AT THE CORRESPONDING ERROR POINTS IS MODIFIED, WOULD BE
17.     00      C USEFUL.)
18.     00      C EQUATION TO BE SOLVED IS
19.     00      C NTYPE(I+1,J+1) * (AX(I,J) + XI(I,J+1) + CX(I,J) + XI(I+2,J+1)
20.     00      C           + AY(I,J) + XI+I,J+1) + XI(I+1,J+2))
21.     00      C           + IZ - NTYPE(I+1,J+1)) * BB(I,J) * XI(I+1,J+1)
22.     00      C           = 2 * (I - NTYPE(I+1,J+1)) * BB(I,J) * XI(I+1,J+1)
23.     00      C           + NTYPE(I+1,J+1) * FI(I,J). ((I=0,I,...,M-1),(J=0,J,...,N-1))
24.     00      C WHERE TERMS OUTSIDE DIMENSION BOUNDARIES ARE INTERPRETED AS ZERO'S.
25.     00      C THUS, IF NTYPE(I,J)=0, XI(I,J) IS LEFT UNCHANGED.
26.     00      MI=M-1
27.     00      NI=N-1
28.     00      M2=M-2
29.     00      N2=N-2
30.     00      NBLKI=NBLK-I
31.     00      DO 80 J=1,N
32.     00      DO 80 I=1,M
33.     00      NTYP(E(I,J)=0.
34.     00      DO 90 J=2,NI
35.     00      DO 90 I=2,M1
36.     00      NTYP(E(I,J)=I
37.     00      DO 2000 JC=2,NI
38.     00      DO 2000 IC=2,MI
39.     00      NTYP(E(IC,JC)=0
40.     00      NTYP(E(IC-I,JC)=0
41.     00      NTYP(E(IC+I,JC)=0
42.     00      NTYP(E(IC,JC-1)=0
43.     00      NTYP(E(IC,JC+1)=0
44.     00      WRITE(6,92) ((NTYP(E(I,J),I=I,M),J=1,N)
45.     01      FORMAT(10X,9III)
46.     00      CALL TOPOLIRECUR(ILFT,IRGT,IGES,JGES,ICHK,JCHK,NEV,LINK,NLINK,
47.     00      A NTYP(E,IE,M,N,M1,N1,M2,NBLK,NBLKI,MXGS,L1,L2,L0J,L02)
48.     00      DO 100 J=1,N2
49.     00      DO 100 I=1,M2
50.     00      AX(I,J)=I.+0I*(I-4)**2
51.     00      CX(I,J)=I.-0I*(I-4)**2
52.     00      AY(I,J)=I.+0I*(J-5)**2
53.     00      BB(I,J)=I-AX(I,J)-AY(I,J)+0I*(J-5)**2-I.I)/(1.-0I*(J-5)
54.     00      A**2)
55.     00      100 CONTINUE
56.     00      DENOM=(.25*M*N)**2
57.     00      DO 170 J=1,N
58.     00      DO 170 I=1,M

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VERSION 3
(continued)

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58.      00      X(1,J)=I(1-1)*(1-M)*J-1.)*I(J-N)/DENOM
59.      00      XX(1,J)=X(1,J)*(1-NTYPE)I,J))
60.      00      170 CONTINUE
61.      00      FSUM=0.
62.      00      DO 171 J=2,N1
63.      00      L=J-1
64.      00      DO 171 I=2,M1
65.      00      K=I-1
66.      00      FX(K,L)=AX(K,L)*X(I,J)+AY(K,L)*X(I,L)+CX(K,L)*X(I+1,J)+*
67.      00      A X(I,J+1)+BB(K,L)*X(I,J)
68.      00      171 FSUM=FSUM+ABS(FX(K,L))
69.      00      FSUM=M2*N2/FSUM
70.      00      T0=SECOND(TIM)
71.      01      CALL JVADAX,AY,BB,CX,RINV,RINV1,H,RECUR,1LFT,1RGT,1GES,JGES,
72.      00      A1CHK,JCHK,NEV,LINK,NLINK,NTYPE,1E,M,N,M2,N2,NBLK,NBLK1,MXGS,L1,L2)
73.      00      T1SUM=0.
74.      00      DO 181 J=2,N1
75.      00      DO 181 I=2,M1
76.      00      OI,I-1,J-1)=F(I-1,J-1)
77.      00      181 X(I,J)=0.
78.      00      ALTHE=0.
79.      00      T0=SECOND(TIM)
80.      01      CALL NORJAX,AY,BB,CX,RINV,RINV1,0UM0,0UM1,0UM2,0,H,XX,RECUR,
81.      00      A 1LFT,1RGT,1GES,JGES,1CHK,JCHK,NEV,LINK,NLINK,1E,M,N,M2,N2,NBLK,
82.      00      B NBLK1,MXGS,L1,L2)
83.      00      DO 990 J=2,N1
84.      00      DO 990 I=2,M1
85.      00      990 X(1,J)=X(1,J)+XX(I,J)
86.      00      DO 991 J=1,N
87.      00      DO 991 I=1,M
88.      00      991 XX(I,J)=0.
89.      00      DO 992 J=2,N1
90.      00      L=J-1
91.      00      DO 992 I=2,M1
92.      00      K=I-1
93.      00      992 OI,K,L)=FX(K,L)-AY(K,L)*X(I,J-1)-AX(K,L)*X(I-1,J)-BB(K,L)*
94.      00      A CX(K,L)*X(I+1,J)-X(I,J+1)
95.      00      1C00 CONTINUE
96.      00      9C5 FORMAT(1X,1P15E8.1)
97.      00      ERSUM=0.
98.      00      DO 740 I=2,M1
99.      00      K=I-1
100.     00      DO 740 J=2,N1
101.     00      L=J-1
102.     00      ERR(I,J)=FX(K,L)-AY(K,L)*X(I,J-1)-AX(K,L)*X(I-1,J)-BB(K,L)*
103.     00      A X(I,J)-CX(K,L)*X(I+1,J)-X(I,J+1)
104.     00      ERR(I,J)=ERR(I,J)+NTYPE(I,J)
105.     00      ERSUM=ERSUM+ABS(ERR(I,J))
106.     00      740 ERR(I,J)=ERR(I,J)+FSUM
107.     00      ERSUM=ERSUM+FSUM/(M2*N2)
108.     00      WRITE(6,904)
109.     00      9C4 FORMAT(1/4GX,20HNORMALIZED RESIDUALS)
110.     03      DO 742 I=2,M1
111.     00      WRITE(6,905)(ERR(I,J),J=2,N1)
112.     00      742 CONTINUE
113.     00      WRITE(6,906) ERSUM
114.     00      9C6 FORMAT(1/37H MEAN ABSOLUTE NORMALIZED RESIDUAL = ,1PE9.2)
115.     00      IF (.ERSUM.GT.1.E-4) STOP
116.     00      IF (I1C.NE.2) NTYPE(I1C-1,JC)=1
117.     03      IF (I1C.NE.M1) NTYPE(I1C+1,JC)=1
118.     00      IF (IJC.NE.2) NTYPE(IJC,JC-1)=1
119.     00      IF (IJC.NE.M1) NTYPE(IJC,JC+1)=1
120.     00      2C00 NTYPE(I1C,JC)=1
121.     00      ENO

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VERSION 3
(continued)

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1.      0I      SUBROUTINE JAYFAX,AY,BB,CX,RINV,RINV1,H,RECUR,ILFT,
2.      00      1IRGT,1GES,JGES,ICHK,JCHK,NEV,LINK,NLINK,NTYPE,1E,MD,ND,M2,N2,
3.      00      A NBLK,NBLK1,MXGS,L1,L2)
4.      00      DIMENSION AX(M2,N2),AY(M2,N2),BB(M2,N2),CX(M2,N2),
5.      00      A RINV(1),RINV1(1),RECUR(M0,NO),H(MD,ND),
6.      00      B ILFT(ND),IRGT(ND),L1(NBLK1),L2(NBLK1),1GES(MXGS,NBLK),
7.      00      C JGES(MXGS,NBLK),ICHK(MXGS,NBLK),JCHK(MXGS,NBLK),NEV(NBLK),
8.      00      O LINK(M2,NBLK1),NLINK(NBLK1),NTYPE(ND,NO),IE(NBLK)
9.      00      JL=1
10.     00      LD=L1(NBLK)
11.     00      NB=0
12.     00      1FO      NB=NB+1
13.     00      NGES=NEV(NR)
14.     00      NGESI=NEV(NB-1)
15.     00      LA=L1(NB)
16.     00      LB=L2(NB-1)
17.     00      LC=L2(NB)
18.     00      JH=IE(NB)
19.     00      JHP=JH+1
20.     00      JHM=JH-2
21.     00      DO 250 NG=1,NGES
22.     00      IG=1GES(NG,NB)
23.     00      JG=1GES(NG,NB)
24.     00      JF=JG-I
25.     00      DO 210 J=JL,JHP
26.     00      DO 210 I=1,MD
27.     00      210 H(I,J)=0.
28.     00      H(IG,JG)=1.
29.     00      IF 3JF.GT.JHM) GO TO 228
30.     00      IF 3NTYPE(1G,JF).EQ.0) GO TO 220
31.     00      NOP=NLINK(NB-1)
32.     00      DO 214 N=1,NOP
33.     00      M=LINK(N,NB-1)
34.     00      IF (ICHK(M,NB-1).EQ.1).AND.ANO.JCHK(M,NB-1).EQ.JF) GO TO 215
35.     00      214 CONTINUE
36.     00      215 DO 218 N=1,NOP
37.     00      L=LINK(N,NB-1)
38.     00      218 H(ICHK(L,NB-1),JF)=RINV1(LB+IN-1)*NGES1*M)
39.     00      220 DO 225 J=JF,JHM
40.     00      IL=ILFT(J+2)-I
41.     00      IR=IRGT(J+2)-1
42.     00      222 DO 225 I=IL,IR
43.     00      225 H(I+1,J+2)=RECUR(I+1,J+2)*(AX(I,J+H(I,J+I))*AY(I,J+H(I+1,J+I))
44.     00      A BB(I,J+H(I+1,J+1)*CX(I,J+H(I+1,J+2),J+I)))
45.     00      220 DO 230 NC=1,NGES
46.     00      1=ICHK(1NC,NB)-I
47.     00      J=JCHK(1NC,NB)-1
48.     00      230 RINV(LA+(NC-1)*NGES+NG)=AX(1,J+H(I,J+I))*AY(1,J+H(I+1,J)+BB(1,J+I)
49.     00      A H(I+1,J+I)*CX(I,J+H(I+2,J+I))
50.     00      1F (NB.EQ.NBLK) GO TO 250
51.     00      NOP=NLINK(NB)
52.     00      J=IE(NB)
53.     00      DO 240 N=1,NOP
54.     00      M=LINK(N,NB)
55.     00      240 RINV(LD+IN-1)*NGES+NG)=H(ICHK(M,NB),J)
56.     00      250 H(IG,JG)=0.
57.     00      CALL MATINV(RINV(LA+1),NGES,NGES)
58.     00      IF 3NB.EQ.NBLK) RETURN
59.     00      DO 260 I=1,NGES
60.     00      DO 260 J=1,NOP
61.     00      RINV1(LC+J-I)*NGES+I)=0.
62.     00      DO 260 K=1,NGES
63.     00      260 RINV1(LC+J-1)*NGES+I)=RINV1(LC+J-1)*NGES+I)-
64.     00      A RINV1(LA+K-1)*NGES+I)*RINV1(LD+(J-1)*NGES+K)
65.     00      C RINV1(1,J,NB) IS THE "ALMOST HOMOGENEOUS" SOLUTION AT THE J-TH OPEN
66.     00      C RESIDUAL POSITION FORCED BY A RESIDUAL VALUE OF 1 AT THE I-TH
67.     00      C RESIDUAL POSITION. HOMOGENEOUS B.C.'S ARE ASSUMED EVERYWHERE,
68.     00      C INCLUDING THE TOP OF THE PRESENT SUBREGION NB.
69.     00      JL=JH
70.     00      GO TO 100
71.     00      END

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VERSION 3
(continued)

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1.      01
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      SUBROUTINE NOR(AX,AY,BB,CX,RINV1,DUMD,DUM1,DUM2,F,H,X,
IRECUR,ILFT,IRGT,IGES,JGES,ICHK,JCHK,NEV,LINK,NLINK,IE,MD,ND,M2,
A,N2,NBLK,NBLK1,NGS,L1,L2)
      DIMENSION AX(M2,N2),AY(M2,N2),BB(M2,N2),CX(M2,N2),
A(RINV1),RINV1(1),RECUR(MD,ND),H(MD,ND),
B(ILFT(ND),IRGT(ND),L1(NBLK),L2(NBLK)),IGES(MGS,NBLK),
C(JGES(MGS,NBLK),JCHK(MGS,NBLK),JCHK(MGS,NBLK),NEV(NBLK)),
D(LINK(M2,NBLK),NLINK(NBLK),IE(NBLK))
      DIMENSION DUMD(M2,NBLK1),DUM1(MGS),DUM2(MGS),F(M2,N2),X(MD,ND)
JS=1
DO 150 NB=1,NBLK
LC=L2(NB)
JF=IE(NB)-2
DO 105 J=JS,JF
IL=ILFT(J-2)-1
IR=IRGT(J-2)-1
DO 105 I=IL,IR
105 X(I+1,J+2)=RECUR(I+I,J+2)*(F(I,J)-BX(I,J)*X(I,J+1)-AY(I,J)*
A(X(I+1,J+1)-BB(I,J)*X(I+1,J+2))-CX(I,J)*X(I+2,J+1))+I.-.
B(RECUR(I+1,J+2))*X(I+1,J+2)
IF (NB.EQ.NBLK) GO TO 150
NGES=NEV(NB)
DO 115 N=1,NGES
I=ICHK(N,NB)-1
J=JCHK(N,NB)-1
115 DUM1(N)=F(I,J)-AX(I,J)*X(I,J+1)-AY(I,J)*X(I+1,J)-BB(I,J)*
A(X(I+1,J+1)-CX(I,J)*X(I+2,J+1)-X(I+1,J+2))
NOP=NLINK(NB)
J=IE(NB)
DO 120 N=1,NOP
DUM2(N)=0.
DO 118 N=1,NGES
118 DUM2(N)=DUM2(N)+DUM1(N)*RINV1(LC+(N-1)*NGES+N)
M=LINK(N,NB)
I=ICHK(M,NB)
DUMD(N,NB)=X(I,J)
120 X(I,J)=X(I,J)-DUM2(N)
150 JS=IE(NB)
DO 300 NB=1,NBLK
NB=NBLK-NB+1
JS=I
IF (NB.NE.I) JS=IE(NB-I)
JF=IE(NB)-2
LA=L1(NB)
LB=L2(NB-1)
NGES=NEV(NB)
IF (NB.EQ.NBLK) GO TO 201
J=IE(NB)
NOP=NLINK(NB)
DO 200 N=1,NOP
M=LINK(N,NB)
I=ICHK(M,NB)
I=ICHK(M,NB)
200 X(I,J)=DUMD(N,NB)
201 CONTINUE
N=IE(NB)
DO 202 J=JS,N
DO 202 I=1,MD

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VERSION 3
(continued)

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58.      00      202  M(I,J)=0.
59.      00      DD 210  N=1,NGES
60.      00      I=ICHK(N,NB)-1
61.      00      J=JCHK(N,NB)-1
62.      00      210  DUM1(N)=F(I,J)-AX(I,J)*X(I,J+1)-AY(I,J)*X(I+1,J)-BB(I,J)*
63.      00      A  X(I+1,J+1)-CX(I,J)*X(I+2,J+1)-X(I+1,J+2)
64.      00      DO 220  N=1,NGES
65.      00      DUM2(N)=0.
66.      00      DO 218  N=1,NGES
67.      00      218  DUM2(N)=DUM2(N)+DUM1(M)*RINV(LA+(N-1)*NGES+M)
68.      00      I=IGES(N,NB)
69.      00      J=JGES(N,NB)
70.      00      H(I,J)=DUM2(N)
71.      00      220  X(I,J)=X(I,J)+DUM2(N)
72.      00      IF (NB.EQ.1) GO TO 250
73.      00      DD 222  N=1,MXGS
74.      00      222  DUM1(N)=0.
75.      00      NOP=NLINK(NB-1)
76.      00      J=JS
77.      00      DO 230  N=1,NOP
78.      00      M=LINK(N,NB-1)
79.      00      I=ICHK(M,NB-1)
80.      00      230  DUM1(M)=H(I,J+I)
C  WHEN BOUNDARY IS IRREGULAR, THERE IS SOME WASTED CALCULATION HERE IN
C  ORDER TO AVOID LOGICAL DECISIONS OR EXTRA STORAGE (GES COULD BE
C  REPLACED BY NLINK(NB))
81.      00      NGES=NEV(NB-1)
82.      00      DD 240  N=1,NOP
83.      00      DUM2(N)=0.
84.      00      DD 238  M=1,NGES
85.      00      238  DUM2(M)=DUM2(N)+DUM1(M)*RINV(LB+(N-1)*NGES+M)
86.      00      M=LINK(N,NB-1)
87.      00      240  H(ICHK(M,NB-1),JCHK(M,NB-1))=DUM2(M)
88.      00      250  DO 370  J=JS,JF
89.      00      IL=ILFT(J+2)-1
90.      00      IR=IRGT(J+2)-1
91.      00      DO 360  I=IL,IR
92.      00      DUM1(I)=RECUR(I+1,J+2)*(AX(I,J)*H(I,J+1)+AY(I,J)*H(I+1,J+1)
93.      00      A  BB(I,J)*H(I+1,J+1)+CX(I,J)*H(I+2,J+1))
94.      00      H(I+1,J+2)=DUM1(I+(I-RECUR(I+1,J+2))*H(I+1,J+2)
95.      00      360  X(I+1,J+2)=X(I+1,J+2)+DUM1(I)
96.      00
97.      00
98.      00
99.      00      END

```

VERSION 3
(continued)

```

1.      01      SUBROUTINE TOPOLIRECUR,ILFT,IRGT,IGES,JGES,ICHK,JCHK,NEV,LINK,
2.      00      INLINK,NTYPE,IE,M,N,M1,M2,NBLK,NBLK1,MXGS,L1,L2,LD1,LD2
3.      00      DIMENSION RECUR(M,N),
4.      00      ILFT(M),IRGT(M),IGES(MXGS,NBLK),
5.      00      A JGES(MXGS,NBLK1),ICHK(MXGS,NBLK1),JCHK(MXGS,NBLK1),NEV(NBLK),
6.      00      BLINK(M2,NBLK1),NLINK(NBLK1),NTYPE(M,N,IE(NBLK1,L1(NBLK1,L2(NBLK1)
7.      00      DO 10 J=2,N1
8.      00      I=0
9.      00      2      I=I+1
10.     00      IF (NTYPE(I,J) .NE. NTYPE(I,J-1).EQ.0.AND.I.LT.M1) GO TO 2
11.     00      ILFT(J)=I
12.     00      I=M
13.     00      4      I=I-1
14.     00      IF (NTYPE(I,J) .NE. NTYPE(I,J-1).EQ.0.AND.I.GT.1) GO TO 4
15.     00      10     IRGT(J)=I
16.     00      IRGT(1)=2
17.     00      ILFT(1)=2
18.     00      ILFT(M)=2
19.     00      IRGT(M)=2
20.     00      DO 15 J=2,N1
21.     00      IF (ILFT(J).LE.IRGTE(J)) GO TO 15
22.     00      IRGT(J)=2
23.     00      ILFT(J)=2
24.     00      15     CONTINUE
25.     00      JMIN=2
26.     00      MXLNK=0
27.     00      L1(1)=0
28.     00      L2(1)=0
29.     00      DO 100 NB=1,NBLK
30.     00      NGES=0
31.     00      NCNK=0
32.     00      JMAX=IE(NB)
33.     00      DO 40 I=2,M1
34.     00      NM=NTYPE(I,JMIN-1)
35.     00      NU=NTYPE(I,JMIN)
36.     00      DO 40 J=JMIN,JMAX
37.     00      NLINK
38.     00      NMNU
39.     00      RECUR(I,J)=NM
40.     00      IF (NL.EQ.0) RECUR(I,J)=0.
41.     00      NU=NTYPE(I,J+1)
42.     00      IF (NM.EQ.0) GO TO 40
43.     00      IF (NL.EQ.1.AND.J.NE.JMIN) GO TO 20
44.     00      NGES=NGES+1
45.     00      IF (NGES.GT.MXGS) GO TO 200
46.     00      IGES(NGES,NB)=I
47.     00      JGES(NGES,NB)=J
48.     00      20     IF (NU.EQ.1.AND.J.NE.JMAX) GO TO 40
49.     00      NCNK=NCNK+1
50.     00      ICHK(NCNK,NB)=I
51.     00      JCHK(NCNK,NB)=J
52.     00      40     CONTINUE
53.     00      NEV(NB)=NGES
54.     00      IF (NB.EQ.NBLK) GO TO 100
55.     00      NLNK=0
56.     00      DO 80 K=1,NGES
57.     00      I=ICHK(K,NB)
58.     00      J=JCHK(K,NB)
59.     00      IF (NTYPE(I,J+1).EQ.0) GO TO 80
60.     00      NLNK=NLNK+1
61.     00      LINN(NLNK,NB)=K
62.     00      80     CONTINUE
63.     00      NLINK(NB)=NLNK
64.     00      MXLNK=MAX0(MXLNK,NLNK+NGES)
65.     00      L1(NB+1)=L1(NB)+NGES*#2
66.     00      IF (NB.NE.NBLK1) L2(NB+1)=L2(NB)+NGES+NLNK
67.     00      LRINV1=L2(NB)+NGES+NLNK
68.     00      C IF MXLNK.GT.NGES+NGES, THERE IS SOME WASTED STORAGE THAT IS VERY
69.     00      C DIFFICULT TO PROGRAM AROUND, BECAUSE OF THE DUAL ROLE OF THE LAST
70.     00      C BLOCK OF RINV, WHICH IS USED AS A SCRATCH AREA FOR RINV1 DATA.
71.     00      C LRINV1=L1(NBLK)+MAX0(MXLNK,NGES*#2)
72.     00      C WRITE(6,150) LPINV,LRINV1
73.     00      150    FORMAT(5H DIMENSIONS REQUIRED FOR RINV AND RINV1 ARE LD1, LD2 = ,
74.     00      AIS,2X,15,I4/124H IF EITHER INPUT VALUE LD1 OR LD2 IS TOO SMALL, T
75.     00      BME COMPUTATION IS HALTED HERE, AND THE USER MUST INCREASE LD1 AND/
76.     00      COR LD2 TO 22M THE INDICATED VALUES.)
77.     00      IF ((LD1.LT.LRINV1).OR.(LD2.LT.LRINV1)) STOP
78.     00      RETURN
79.     00      200    WRITE(6,202)
80.     00      202    FORMAT(92H DIMENSION PARAMETER MXGS IS TOO SMALL FOR SPECIFIED TOP
81.     00      AGOGRAPHY AND SUBREGION SPECIFICATION.)
82.     00      STOP
83.     00      END

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VERSION 3
(continued)

```

1.      00      SUBROUTINE MATINVIR,N,M)
2.      00      DIMENSION B(N,1),B1(100),B2(100)
3.      00      M1=M-1
4.      00      DO 110 I=1,M1
5.      00      B1(I,1)=1./B(1,1)
6.      00      B(1,1)=1.0
7.      00      DO 112 J=1,M
8.      00      B41,J)=B(1,J)*B1(1)
9.      00      112   IP1=I+1
10.     00      DO 120 I1=IP1,M
11.     00      B1(I1)=B41,1)
12.     00      DO 125 I1=IP1,M
13.     00      B41,1)=0.
14.     00      DO 127 J=1,M
15.     00      B2(J)=B41,J)
16.     00      DO 135 I1=IP1,M
17.     00      DO 135 J=1,M
18.     00      B411,J)=B(I1,J)-B1(I1)*B2(J)
19.     00      135   CONTINUE
20.     00      B1(1)=1./B(M,M)
21.     00      B(M,M)=1.
22.     00      DO 140 J=1,M
23.     00      B(M,J)=B(M,J)*B1(1)
24.     00      DO 150 I2=1,M
25.     00      DO 155 I2=1,1
26.     00      B1(12)=B412,1)
27.     00      IM1=1-1
28.     00      DO 156 I2=1,IM1
29.     00      B412,1)=0.
30.     00      DO 157 J=1,M
31.     00      B2(J)=B1,J)
32.     00      IM1=1-1
33.     00      DO 160 I2=1,IM1
34.     00      DO 160 J=1,M
35.     00      B412,J)=B(I2,J)-B1(I2)*B2(J)
36.     00      160   CONTINUE
37.     00      END

```

VERSION 4

```

1.      00      PROGRAM MAIN(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
2.      00      COMMON/SEVP/AX(7,9),AY(7,9),BB(7,9),CX(7,9),0(7,9),F(7,9),
3.      00      A(RINV1$08),RINV1(308),QUMD114,21,0UM11141,
4.      00      B(DUM2(14),X(9,11),M(9,11),ERR(9,11),XX(9,11),ILFT(11),
5.      00      C(IRGT(11),IGES(14,3),JGES(14,3),ICHK(14,3),JCHK(14,3),NEV(3),
6.      00      D(LINK(7,2),NLINK(21,NTYPE(9,11),IE(3),LI(3),L2(2)
7.      00      DATA M,N,MXGS,NBLK,L0I,L02/9,11,14,3,588,308/
8.      00      DATA IE/4,7,16/
9.      00
10.     00      C DIETRICH'S MODIFIED MAOLAL SOLVER, VERSION 4.
11.     00      C THIS VERSION IS IDENTICAL TO VERSION 3, EXCEPT
12.     00      C NO ISLANDS ARE ALLOWED AND IRREGULAR BOUNDARIES MUST BE SUCH THAT ALL
13.     00      C NONZERO NTYPE VALUES IN A GIVEN ROW MUST BE CONTIGUOUS, THEREBY
14.     00      C REQUIRING THAT THE RIGHT AND LEFT BOUNDARY CURVES BE SINGLE VALUED
15.     00      C FUNCTIONS OF Y. THIS ELIMINATES THE NEED FOR ARRAY RECUR AND
16.     00      C ASSOCIATED COMPUTATIONS.
17.     00      C EQUATION TO BE SOLVED IS
18.     00      C   NTYPE(I+1,J+1) * (AX(I,J) + X(I,J+1) + CX(I,J) + X(I+2,J+1)
19.     00      C           + AY(I,J) + X(I+1,J+1) + X(I+1,J+2)) +
20.     00      C           + (2 - NTYPE(I+1,J+1)) * BB(I,J) + X(I+1,J+1)
21.     00      C           = 2 * (1 - NTYPE(I+1,J+1)) * BB(I,J) + X(I+1,J+1)
22.     00      C           + NTYPE(I+1,J+1) * F(I,J), ((I=0,I,...,M-1),(J=0,1,...,N-1))
23.     00      C WHERE TERMS OUTSIDE DIMENSION BOUNDS ARE INTERPRETED AS ZERO'S.
24.     00      C THUS, IF NTYPE(I,J)=0, X(I,J) IS LEFT UNCHANGED.
25.     00      M1=M-1
26.     00      N1=N-1
27.     00      M2=M-2
28.     00      N2=N-2
29.     00      NBLK1=NBLK-1
30.     00      DO 80 J=1,N
31.     00      DO 80 I=1,M
32.     00      8C  NTYPE(I,J)=0.
33.     00      DO 90 J=2,N1
34.     00      9C  NTYPE(I,J)=1
35.     01      NTYPE(12,3),NTYPE(2,4),NTYPE(3,4),NTYPE(2,5),NTYPE(3,5),NTYPE(2,6)
36.     00      A=.0
37.     00      WRITE(6,92) ((NTYPE(I,J),I=1,M),J=1,N)
38.     00      92  FORMA1(IDX,971)
39.     02      CALL TOPOL(ILFT,IRGT,IGES,JGES,ICHK,JCHK,NEV,LINK,NLINK,
40.     00      A,NTYPE,IE,M,N,M1,N1,M2,NBLK,NBLK1,MXGS,L1,L2,L0I,L02)
41.     00      DO 100 J=1,N2
42.     00      DO 100 I=1,M2
43.     00      AX(I,J)=1.0*(I-1)**2
44.     00      CX(I,J)=1.0*(I-1)**2
45.     00      AY(I,J)=1.0*(J-1)**2
46.     00      BB(I,J)=-(AX(I,J)-AY(I,J)-CX(I,J)+.01*(J-5)**2-I-1)/(1.0*(J-5)
47.     00      A**2)
48.     00
49.     00      100  CONINUE
50.     00      DENOM=1.25*M*N)**2
51.     00      DO 170 J=1,N
52.     00      DO 170 I=1,M
53.     00      X(I,J)=(I-1)*(I-M)*(J-1)*(J-N)/DENOM
54.     00      XX(I,J)=X(I,J)*(-NTYPE(I,J))
55.     00      170  CONTINUE
56.     00      FSUM=0.
57.     00      DO 171 J=2,N1
58.     00      L=J-1

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VERSION 4
(continued)

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58.      00      DO I7I I=2,NI
59.      00      K=I-I
60.      00      F(I,K,L)=AX(K,L)*X(I,J)+AY(K,L)*X(I,J)+CX(K,L)*X(I,J)+
61.      00      A X(I,J+1)*RB(K,L)*X(I,J)
62.      00      FSUM=FSUM+ABS(F(I,K,L))
63.      00      FSUM=M2*N2/FSUM
64.      00      TO=SECOND(TIM)
65.      02      CALL JAYIA(X,AY,BB,CX,RINV,RINV1,M,ILFI,IRGT,IGES,JGES,
66.      00      AICHK,JCHK,NEV,LINK,NLINK,NTYPE,IE,M,N,M2,N2,NBLK,NBLKI,MGES,L1,L2)
67.      00      TIMSUM=0.
68.      00      DO I8I I=2,NI
69.      00      DO I8I I=2,MI
70.      00      OII-I,J-I=F(I-I,J-I)
71.      00      X1,I,J=0.
72.      00      ALINE=0.
73.      00      TO=SECOND(TIM)
74.      02      CALL NORIA(X,AY,BB,CX,RINV,RINV1,DUM0,DUM1,DUM2,0,M,XX,
75.      00      A ILFT,IRGI,IGES,JGES,ICHK,JCHK,NEV,LINK,NLINK,IE,M,N,M2,N2,NBLK,
76.      00      B NBLKI,MGES,L1,L2)
77.      00      DO 990 J=2,NI
78.      00      DO 990 I=2,MI
79.      00      X1,I,J=X1,I,J+XX(I,J)
80.      00      DO 991 J=1,N
81.      00      DO 991 I=1,M
82.      00      991 XX(I,J)=0.
83.      00      DO 992 J=2,NI
84.      00      L=J-I
85.      00      DO 992 I=2,MI
86.      00      K=I-J
87.      00      992 OIK,L=F(I,K,L)-AY(K,L)*X(I,J-I)-AX(K,L)*X(I-I,J)-BB(K,L)*X(I,J)-
88.      00      A CX(K,L)*X(I,J+I)
89.      00      1E00 CONTINUE
90.      00      9CS FORMAI(IX,IPISE8.II
91.      00      ERSUM=0.
92.      00      DO 740 I=2,MI
93.      00      K=I-1
94.      00      DO 740 J=2,NI
95.      00      L=J-I
96.      00      ERR(I,J)=F(I,K,L)-AY(K,L)*X(I,J-I)-AX(K,L)*X(I-I,J)-BB(K,L)*
97.      00      A X(I,J)-CX(K,L)*X(I,J-I)-X(I,J+I)
98.      00      ERR(I,J)=ERR(I,J)*NTYPE(I,J)
99.      00      ERSUM=ERSUM+ABS(ERR(I,J))
100.     00      740 ERR(I,J)=ERR(I,J)*FSUM
101.     00      ERSUM=ERSUM+FSUM/(M2*N2)
102.     00      WRITE(6,904)
103.     00      9C4 FORMAI(/NDX,20NNORMALIZED RESIDUALS)
104.     00      DO 742 I=2,MI
105.     00      WRITIE(6,905)(ERR(I,J),J=2,NI)
106.     00      742 CONTINUE
107.     00      WRITE(6,906) ERSUM
108.     00      9C6 FORMAT(1/37H MEAN ABSOLUTE NORMALIZED RESIDUAL = ,IPE9.2)
109.     00      IF (ERSUM.GT.1.E-4) SIOP
110.     00      ENO

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VERSION 4
(continued)

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1.      01      SUBROUTINE JAYIA(X,AY,BB,CX,RINV1,H,ILFT,
2.      00      1IRGT,IGES,ICHK,JCHK,NEV,LINK,NLINK,NTYPE,IE,ND,ND,M2,N2,
3.      00      A NBLK,NBLK1,MXGS,L1,L2)
4.      00      DIMENSION AX(M2,N2),AY(M2,N2),BB(M2,N2),CX(M2,N2),
5.      00      A RINV(1),RINV1(1),H(ND,ND),
6.      00      B ILFT(ND),IRGT(ND),L1(NBLK),L2(NBLK1),IGES(MXGS,NBLK),
7.      00      C JGES(MXGS,NBLK),JCHK(MXGS,NBLK),JCHK(MXGS,NBLK),NEV(NBLK),
8.      00      D LINK(M2,NBLK1),NLINK(NBLK1),NTYPE(ND,ND),IE(NBLK)
9.      00      JL=1
10.     00      LD=L1(NBLK)
11.     00      NB=0
12.     00      ICD NB=NB+1
13.     00      NGES=NEV(NB)
14.     00      NGES1=NEV(NB-1)
15.     00      LA=L1(NB)
16.     00      LB=L2(NB-1)
17.     00      LC=L2(NB)
18.     00      JH=IE(NB)
19.     00      JHP=JH+1
20.     00      JHM=JH-2
21.     00      DO 250 NG=1,NGES
22.     00      IG=IGES(NG,NB)
23.     00      JG=JGES(NG,NB)
24.     00      JF=JG-1
25.     00      DO 210 J=JL,JHP
26.     00      DO 210 I=I,ND
27.     00      210 H(I,J)=0.
28.     00      H(16,JG)=1.
29.     00      IF (JF,GT,JHM) GO TO 220
30.     00      IF (NTYPE(1G,JF).EQ.0) GO TO 220
31.     00      NOP=NLINK(NB-1)
32.     00      DO 214 NI=I,NOP
33.     00      M=LINK(N,ND-1)
34.     00      IF (ICHK(M,ND-1).EQ.IG.AND.JCHK(M,NB-1).EQ.JF) GO TO 215
35.     00      CONTINUE
36.     00      215 DO 218 NI=I,NOP
37.     00      L=LINK(N,ND-1)
38.     00      218 H(I,ICHK(L,ND-1),JF)=RINV1(LB+(N-I)*NGES1+M)
39.     00      220 DO 225 J=JF,JHM
40.     00      IL=ILFT(J+2)-I
41.     00      IR=IRGT(J+2)-I
42.     00      DO 225 I=IL,IR
43.     00      225 H(I+1,J+2)=-(AX(I,J)*H(I,J+1)+AY(I,J)*H(I+1,J)+  
44.     00      A BB(I,J)*H(I+1,J+1)+CX(I,J)*H(I+2,J+1))
45.     00      DO 230 NC=1,NGES
46.     00      I=ICHK(1C,NB)-1
47.     00      J=JCHK(1C,NB)-1
48.     00      230 RINV(LA+(NC-1)*NGES+NG)=AX(I,J)*H(I,J+1)+AY(I,J)*H(I+1,J)+BB(I,J)*  
49.     00      A H(I+1,J+1)+CX(I,J)*H(I+2,J+1)
50.     00      IF (NB.EQ.NBLK) GO TO 250
51.     00      NOP=NLINK(NB)
52.     00      J=IE(NB)
53.     00      DO 240 NI=1,NOP
54.     00      M=LINK(N,ND)
55.     00      240 RINV(LD+(N-1)*NGES+NG)=H(I,ICHK(M,NB),J)
56.     00      250 H(1G,JG)=0.
57.     00      CALL MATINV(RINV(LA+1),NGES,NGES)
58.     00      IF (NB.EQ.NBLK) RETURN
59.     00      DO 260 I=1,NGES
60.     00      DO 260 J=1,NOP
61.     00      RINV1(LC*(J-1)*NGES+I)=0.
62.     00      DO 260 K=1,NGES
63.     00      260 RINV1(LC*(J-1)*NGES+I)=RINV1(LC*(J-1)*NGES+I)-  
64.     00      A RINV(LA*(K-1)*NGES+I)*RINV(LD+(J-1)*NGES+K)
65.     00      C RINV1(1,J,ND) IS THE "ALMOST HOMOGENEOUS" SOLUTION AT THE J-TH OPEN
66.     00      C RESIDUAL POSITION FORCED BY A RESIDUAL VALUE OF 1 AT THE 1-TH
67.     00      C RESIDUAL POSITION. HOMOGENEOUS B.C.'S ARE ASSUMED EVERYWHERE,
68.     00      C INCLUDING THE TOP OF THE PRESENT SUBREGION NB.
69.     00      JL=JH
70.     00      GO TO 100
71.     00      END

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VERSION 4
(continued)

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1.      01      SUBROUTINE NOR1(A,X,BB,CX,RINV1,DUM0,DUM1,DUM2,F,H,X,
2.      00      11LFT,IRGT,IGES,JGES,ICHK,JCHK,NEV,LINK,NLINK,IE,MD,ND,M2,
3.      00      A,N2,NBLK,NBLK1,MXGS,L1,L2)
4.      00      DIMENSION AX(M2,N2),AY(M2,N2),BB(M2,N2),CX(M2,N2),
5.      00      A RINV1(1),RINV1(1),H(MD,ND),
6.      00      8 1LFT(ND),IRGT(ND),L1(NBLK1),IGES(MXGS,NBLK1,
7.      00      C JGES(MXGS,NBLK1),JCHK(MXGS,NBLK1),JCHK(MXGS,NBLK1),NEV(NBLK1),
8.      00      0 LLINK(M2,NBLK1),NLINK(NBLK1),IE(NBLK1)
9.      00      DIMENSION DUM0(M2,NBLK1),DUM1(MXGS),DUM2(MXGS),F(M2,N2),X(MD,ND)
10.     00      JS=1
11.     00      DO 150 NB=1,NBLK
12.     00      LC=L2(NB)
13.     00      JF=IE(NB)-2
14.     00      DO 105 J=JS,JF
15.     00      1L=1LFT(J+2)-1
16.     00      IR=IRGT(J+2)-1
17.     00      DO 105 I=1L,IR
18.     00      1C5 X(I+1,J+2)=F(I,J)-AX(I,J)*X(I,J+1)-AY(I,J)*X(I+1,J)-BB(I,J)*
19.     00      A X(I+1,J+1)-CX(I,J)*X(I+2,J+1)
20.     00      IF (NB.EQ.NBLK) GO TO 150
21.     00      NGES=NEV(NB)
22.     00      DO 115 N=1,NGE5
23.     00      I=ICHK(N,NB)-1
24.     00      J=JCHK(N,NB)-1
25.     00      1I5 DUM1(N)=F(I,J)-AX(I,J)*X(I,J+1)-AY(I,J)*X(I+1,J)-BB(I,J)*
26.     00      A X(I+1,J+1)-CX(I,J)*X(I+2,J+1)-X(I,J+2)
27.     00      NOP=NLINK(NB)
28.     00      J=IE(NB)
29.     00      DO 120 N=1,NOP
30.     00      DUM2(N)=0.
31.     00      DO 118 M=1,NGE5
32.     00      1I8 DUM2(N)=DUM2(N)+DUM1(M)*RINV1(LC+(N-1)*NGE5+M)
33.     00      M=LINK(N,NB)
34.     00      I=ICHK(M,NB)
35.     00      DUM0(N,NB)=X(I,J)
36.     00      120 X(I,J)=X(I,J)-DUM2(M)
37.     00      150 JS=IE(NB)
38.     00      DO 300 NB1=1,NBLK
39.     00      NB=NBLK-NB1+1
40.     00      JS=1
41.     00      IF (NB.NE.I) JS=IE(NB-1)
42.     00      JF=IE(NB)-2
43.     00      LA=L1(NB)
44.     00      LB=L2(NB-1)
45.     00      NGES=NEV(NB)
46.     00      IF (NB.EQ.NBLK) GO TO 201
47.     00      J=IE(NB)
48.     00      NOP=NLINK(NB)
49.     00      DO 200 N=1,NOP
50.     00      M=LINK(N,NB)
51.     00      I=ICHK(M,NB)
52.     00      2C0 X(I,J)=DUM0(N,NB)
53.     00      2E1 CONTINUE
54.     00      N=IE(NB)
55.     00      DO 202 J=JS,N
56.     00      DO 202 I=1,MD

```

VERSION 4
(continued)

```

57.    00      202 M(I,J)=0.
58.    00      DO 210 N=I,NGES
59.    00      I=ICHK(N,NB)-1
60.    00      J=JCHK(N,NB)-1
61.    00      210 DUM1(N)=F(I,J)-AX(I,J)*X(I,J+1)-AY(I,J)*X(I+1,J)-BB(I,J)*
62.    00      A X(I+1,J+1)-CX(I,J)*X(I+2,J+1)-X(I+1,J+2)
63.    00      DO 220 N=I,NGES
64.    00      DUM2(N)=0.
65.    00      DO 230 M=1,NGES
66.    00      210 DUM2(N)=DUM2(N)+DUM1(M)*RINV(LA+(N-I)*NGES+M)
67.    00      I=IGES(N,NB)
68.    00      J=JGES(N,NB)
69.    00      H(I,J)=DUM2(N)
70.    00      220 X(I,J)=X(I,J)+DUM2(N)
71.    00      IF (NB.EQ.I) GO TO 250
72.    00      DO 222 N=1,MXGS
73.    00      222 DUM1(N)=0.
74.    00      NOP=NLINK(NB-I)
75.    00      J=JS
76.    00      DO 230 N=I,NOP
77.    00      M=LINK(N,NB-I)
78.    00      I=ICHK(M,NB-I)
79.    00      230 DUM1(M)=H(I,J+1)
80.    00      C WHEN BOUNDARY IS IRREGULAR, THERE IS SOME WASTED CALCULATION HERE IN
81.    00      C ORDER TO AVOID LOGICAL DECISIONS OR EXTRA STORAGE (GES COULD BE
82.    00      C REPLACED BY NLINK(NB))
83.    00      NGES=NEVNBNB-I)
84.    00      DO 240 N=I,NOP
85.    00      DUM2(N)=0.
86.    00      DO 238 M=I,NGES
87.    00      238 DUM2(N)=DUM2(N)+DUM1(M)*RINV(LB+(N-I)*NGES+M)
88.    00      M=LINK(N,NB-I)
89.    00      H(ICHK(M,NB-I),JCHK(M,NB-I))=DUM2(N)
90.    00      240 DO 300 J=JS,JF
91.    00      IL=ILFT(J+2)-I
92.    00      IR=IRGT(J+2)-I
93.    00      DO 300 I=IL,IR
94.    00      H(I+1,J+2)=-((AX(I,J)*H(I,J+1)+AY(I,J)*H(I+1,J)+
95.    00      A BB(I,J)*H(I+1,J+1)+CX(I,J)*H(I+2,J+1))
96.    00      300 X(I+1,J+2)=X(I+1,J+2)+H(I+1,J+2)
97.    00      ENO

```

VERSION 4
(continued)

```

1.      01      SUBROUTINE TOPOL(ILFT,IRGT,IGES,JGES,ICHK,JCHK,NEV,LINK,
2.      00      INLINK,NTYPE,IE,M,N,M1,M2,NBLK,NBLK),MXGS,L1,L2,L01,L02)
3.      00      DIMENSION ILFT(N),IRGT(N),IGES(MXGS,NBLK),
4.      00      A_JGES(MXGS,NBLK),ICHK(MXGS,NBLK),JCHK(MXGS,NBLK),NEV(NBLK),
5.      00      BLINK(M2,NBLK),NLINK(NBLK),NTYPE(M,N),IE(NBLK),L1(NBLK),L2(NBLK)
6.      00      DO 10 J=2,NI
7.      00      I=0
8.      00      2      I=I+1
9.      00      IF (NTYPE(I,J)*NTYPE(I,J-1).EQ.0.AND.I.LT.M1) GO TO 2
10.     00      ILFT(J)=I
11.     00      I=M
12.     00      4      I=I-1
13.     00      IF (NTYPE(I,J)*NTYPE(I,J-1).EQ.0.AND.I.GT.1) GO TO 4
14.     00      IRGT(I)=I
15.     00      IRGT(I)=2
16.     00      ILFT(1)=2
17.     00      ILFT(N)=2
18.     00      IRGT(N)=2
19.     00      00 IS J=2,NI
20.     00      IF (ILFT(J).LE.IRG(T(J))) GO TO 15
21.     00      IRGT(J)=2
22.     00      ILFT(J)=2
23.     00      15      CONTINUE
24.     00      JMIN=2
25.     00      MXLNK=0
26.     00      L1(I)=0
27.     00      L2(I)=0
28.     00      DO 100 NB=1,NBLK
29.     00      NGES=0
30.     00      NCHK=0
31.     00      JMAX=IE(NB)
32.     00      DO 40 I=2,M1
33.     00      NM=NTYPE(I,JMIN-1)
34.     00      NU=NTYPE(I,JMIN)
35.     00      DO 40 J=JMIN,JMAX
36.     00      NL=NM
37.     00      NM=NU
38.     00      NU=NTYPE(I,J+1)
39.     00      IF (NM.EQ.0) GO TO 40
40.     00      IF (NL.EQ.0.AND.J.NE.JMIN) GO TO 20
41.     00      NGES=NGES+1
42.     00      IF (NGES.GT.MXGS) GO TO 200
43.     00      IGES(NGES,NB)=I
44.     00      JGES(NGES,NB)=J
45.     00      20      IF (NU.EQ.0.AND.J.NE.JMAX) GO TO 40
46.     00      NCHK=NCHK+1
47.     00      ICHK(NCHK,NB)=I
48.     00      JCHK(NCHK,NB)=J
49.     00      40      CONTINUE
50.     00      NEV(NB)=NGES
51.     00      IF (NB.EQ.NBLK) GO TO 100
52.     00      NLNK=0
53.     00      DO 80 K=1,NGES
54.     00      I=ICHK(K,NB)
55.     00      J=JCHK(K,NB)
56.     00      IF (NTYPE(I,J+1).EQ.0) GO TO 80
57.     00      NLNK=NLNK+1
58.     00      LINK(NLNK,NB)=K
59.     00      80      CONTINUE
60.     00      NLINK(NB)=NLNK
61.     00      MXLNK=MAX0(MXLNK,NLNK+NGES)
62.     00      L1(NB+1)=L1(NB)+NGES**2
63.     00      IF (NB.NE.NBLK) L2(NB+1)=L2(NB)+NGES+NLNK
64.     00      LRINV=L2(NB)+NGES+NLNK
65.     00      100     JMIN=JMAX+1
66.     00      C  IF MXLNK.GT.NGES+NGES, THERE IS SOME WASTED STORAGE THAT IS VERY
67.     00      C  DIFFICULT TO PROGRAM AROUND, BECAUSE OF THE DUAL ROLE OF THE LAST
68.     00      C  BLOCK OF RINV, WHICH IS USED AS A SCRATCH AREA FOR RINV1 DATA.
69.     00      C  LRINV=LRINV+MAX0(MXLNK,NGES**2)
70.     00      WRITE(6,150) LRINV,LRINV1
71.     00      150     FORMAT(5SH DIMENSIONS REQUIRED FOR RINV AND RINV1 ARE L01, L02 = ,
72.     00      A15,2X,I5.1H./124H IF EITHER INPUT VALUE L01 OR L02 IS TOO SMALL, T
73.     00      BHE COMPUTATION IS HALTED HERE, AND THE USER MUST INCREASE L01 AND/
74.     00      COR L02 TO/2H THE INDICATED VALUES.)
75.     00      IF ((L01.LT.LRINV).OR.(L02.LT.LRINV1)) STOP
76.     00      RETURN
77.     00      200     WRITE(6,202)
78.     00      202     FORMAT(92H DIMENSION PARAMETER MXGS IS TOO SMALL FOR SPECIFIED TO-
79.     00      PGRAPHY AND SUBREGION SPECIFICATION.)
80.     00      STOP
81.     00      END

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VERSION 4
(continued)

```

1.    00      SUBROUTINE MATINV(B,N,M)
2.    00      DIMENSION B(N,1),B1(100),B2(100)
3.    00      M1=M-1
4.    00      DO 110 I=1,M1
5.    00      B1(I)=1./B(I,1)
6.    00      B(I,1)=1.0
7.    00      DO 112 J=1,M
8.    00      B(I,J)=B(I,J)*B1(I)
9.    00      IPI=I+1
10.   00      DO 120 II=IPI,M
11.   00      B1(II)=B(II,I)
12.   00      DO 125 II=IPI,M
13.   00      B(I,II)=0.
14.   00      DO 127 J=1,M
15.   00      B2(J)=B(I,J)
16.   00      DO 135 II=IPI,M
17.   00      DO 135 J=1,M
18.   00      B(I,J)=B(I,J)-B1(II)*B2(J)
19.   00      110 CONTINUE
20.   00      B(I,1)=1./B(M,M)
21.   00      B(M,M)=1.
22.   00      DO 140 J=1,M
23.   00      B(M,J)=B(M,J)*B1(I)
24.   00      DO 150 I=2,M
25.   00      DO 155 I2=1,I
26.   00      B1(I2)=B(I2,I)
27.   00      IM1=I-1
28.   00      DO 156 I2=1,IM1
29.   00      B(I2,I)=0.
30.   00      DO 157 J=1,M
31.   00      B2(J)=B(I,J)
32.   00      IM1=I-1
33.   00      DO 160 I2=1,IM1
34.   00      DO 160 J=1,M
35.   00      B(I2,J)=B(I2,J)-B1(I2)*B2(J)
36.   00      150 CONTINUE
37.   00      END

```

VERSION 5

```

1.      02      PROGRAM JAYNOR,INPUT1,OUTPUT,TAPES=INPUT,TAPE6=OUTPUT
2.      00      DIMENSION AX(7,9),AT(7,9),BB(7,9),CX(7,9),O(7,9),F(7,9),
3.      00      A RTNV(7,7,3),RINV(7,7,2),OUM(7,2),OUM1(7),OUM2(7),X(9,11),
4.      00      8 M(9,11),ERR(9,11),XX(9,11),IE(3)
5.      00      DATA M,N,NBLK/9,11,3/
6.      00      DATA IE/4,7,10/
7.      00      C DICTRCH'S MODIFIED MADALA SOLVER, VERSION 5.
8.      00      C THIS VERSION IS IDENTICAL TO VERSION 4, EXCEPT
9.      00      C IRREGULAR BOUNDARIES ARE NOT ALLOWED. IT IS ALSO EQUIVALENT TO
10.     00      C MADALA'S BASIC SOLVER, ALLOWING NON-SEPARABLE OPERATORS ON A
11.     00      C NON-UNIFORM MESH WITH REGULAR BOUNDARIES.
12.     00      C THE EQUATION TO BE SOLVED IS
13.     00      C AX(I,J) * X(I,J+1) + CX(I,J) * X(I+2,J+1) + BB(I,J) * X(I+1,J+1) +
14.     00      C AV(I,J) * X(I+1,J) + X(I+1,J+2) = F(I,J),
15.     00      C ((I=1,M-2),J=1,N-2))
16.     00      C WHERE X(I,J), X(M,J), X(I,1), AND X(I,N) ARE LEFT UNCHANGED.
17.     00      M1=M-1
18.     00      N1=N-1
19.     00      M2=M-2
20.     00      N2=N-2
21.     00      NBLK1=NBLK-1
22.     00      DO 100 J=1,N2
23.     00      DO 100 I=1,M2
24.     00      AX(I,J)=1.+01*(J-1)**2
25.     00      CX(I,J)=1.-01*(J-1)**2
26.     00      AV(I,J)=1.+01*(J-5)**2
27.     00      BB(I,J)=-(AX(I,J)-AV(I,J))-CX(I,J)+01*(J-5)**2-1.1/
28.     00      A (I,-01*(J-5)**2)
29.     00      100 CONTINUE
30.     00      DENOM=2.5*M*N**2
31.     00      DO 170 J=1,N
32.     00      DO 170 I=1,M
33.     00      XX(I,J)=J1-1.01*(J-1)*01*(J-N)/DENOM
34.     00      XX(I,J)=0.
35.     00      IF J1.EQ.1.OR.J1.EQ.M.OR.J.EQ.I.OR.J.EQ.N1 XX(I,J)=X(I,J)
36.     00      170 CONTINUE
37.     00      FSUM=0.
38.     00      DO 171 J=2,N1
39.     00      L=J-1
40.     00      DO 171 I=2,M1
41.     00      K=I-1
42.     00      FIK,L=AX(K,L)*X(I,J)+AV(K,L)*X(I,L)+CX(K,L)*X(I+1,J)+
43.     00      A X(I,J+1)*BB(K,L)*X(I,J)
44.     00      171 FSUM=FSUM+ABS(FIK,K,L))
45.     00      FSUM=M2*N2/FSUM
46.     00      T0=SECOND(JIM)
47.     02      CALL JAYJAX,AT,BB,CX,RINV,RINV1,M,TE,M,N,M2,N2,NBLK,NBLK1)
48.     00      TIPSUM=0.
49.     00      DO 181 J=2,N1
50.     00      DO 181 I=2,M1
51.     00      OJ1-1,J-1=FJ1-1,J-1
52.     00      181 XIT,J=0.
53.     00      ALINE=0.
54.     00      T0=SECOND(JIM)
55.     02      CALL NORJAX,AT,BB,CX,RINV,RINV1,OUM,OUM1,OUM2,O,M,XX,IE,M,N,M2,
56.     00      A N2,NBLK,NBLK1)
57.     00      DO 990 J=2,N1
58.     00      DO 990 I=2,M1
59.     00      XX(I,J)=XX(I,J)+XX(I,J)
60.     00      DO 991 J=1,N
61.     00      DO 991 I=1,M
62.     00      991 XX(I,J)=0.
63.     00      DO 992 J=2,N1
64.     00      L=J-1
65.     00      DO 992 I=2,M1
66.     00      K=I-1
67.     00      992 OJM,L=FIK,L)-AT(K,L)*X(I,J-1)-AX(K,L)*X(I-1,J)-BB(K,L)*X(I,J)+
68.     00      A CX(K,L)*X(I+1,J)-X(I,J)
69.     00      1800 CONTINUE
70.     00      9E5  FORMAT(1X,1P15E8.1)
71.     00      ERSUM=0.
72.     00      DO 780 I=2,M1
73.     00      K=I-1
74.     00      DO 780 J=2,N1
75.     00      L=J-1
76.     00      ERR(I,J)=FIK,L)-AT(K,L)*X(I,J-1)-AX(K,L)*X(I-1,J)-BB(K,L)*X(I,J)+
77.     00      A CX(K,L)*X(I+1,J)-X(I,J)
78.     00      ERSUM=ERSUM+ABS(ERR,I,J))
79.     00      780 ERR(I,J)=ERR,I,J+FSUM
80.     00      ERSUM=ERSUM+FSUM/(M2*N2)
81.     00      WRITE(6,904)
82.     00      904 FORMAT(1/4D,20HNORMALIZED RESIDUALS)
83.     00      DO 742 I=2,M1
84.     00      WR1TE(6,905)ERR,I,J,J=2,N1)
85.     00      742 CONTINUE
86.     00      WR1TE(6,906) ERSUM
87.     00      9E6  FORMAT(1/3D,10H MEAN ABSOLUTE NORMALIZED RESIDUAL = ,1PE9.2)
88.     00      END

```

VERSION 5
(continued)

```

1.      D1      SUBROUTINE JAY(AX,AY,BB,CX,RINV1,M,IE,MD,ND,M2,N2,NBLK,
2.      00      INBLK1)
3.      00      DIMENSION AX(M2,N2),AY(M2,N2),BB(M2,N2),CX(M2,N2),RINV(M2,M2,NBLK)
4.      00      A,RINV1(M2,M2,NBLK1),H(MD,ND),IE(NBLK)
5.      00      JL=I
6.      00      NB=0
7.      00      1CO      NB=N8+1
8.      00      JL=IE(NB)
9.      00      JHP=JH+1
10.     00      JHM=JH-2
11.     00      JG=JL+I
12.     00      DO 250 NG=1,M2
13.     00      IG=NG+1
14.     00      DO 210 J=JL,JHP
15.     00      DO 210 I=1,MD
16.     00      210      H(I,J)=0.
17.     00      H(IG,JG)=I.
18.     00      IF (NB.EQ.I) GO TO 220
19.     00      DO 218 N=1,M2
20.     00      218      H(N+I,JL)=RINV1(NG,N,NB-1)
21.     00      220      DO 225 J=JL,JHM
22.     00      DO 225 I=1,M2
23.     00      225      H(I+1,J+2)=-(AX(I,J)*H(I,J+1)+AY(I,J)*H(I+1,J)+BB(I,J)*H(I+1,J+1) +
A CX(I,J)*H(I+2,J+1))
24.     00      J=JH-1
25.     00      DO 230 I=1,M2
26.     00      230      RINV(NG,1,NB)=AX(I,J)*H(I,J+1)+AY(I,J)*H(I+1,J)+BB(I,J)+
A H(I+1,J+1)*CX(I,J)*H(I+2,J+1)
27.     00      IF (NB.EQ.NBLK) GO TO 250
28.     00      J=IE(NB)
29.     00      DO 240 N=1,M2
30.     00      240      RINV(NG,N,NBLK)=H(N+I,J)
31.     00      250      H(IG,JG)=0.
32.     00      CALL MAINV(RINV1,I,NB),M2,M2)
33.     00      IF (NB.EQ.NBLK) RETURN
34.     00      DO 260 I=1,M2
35.     00      DO 260 J=1,M2
36.     00      RINV1(I,J,NB)=D.
37.     00      DO 260 K=1,M2
38.     00      260      RINV1(I,J,NB)=RINV1(I,J,NB)-RINV(I,K,NB)*RINV(K,J,NBLK)
39.     00      C RINV1(I,J,NB) IS THE "ALMOST HOMOGENEOUS" SOLUTION AT THE J-TH OPEN
40.     00      C RESIDUAL POSITION FORCED BY A RESIDUAL VALUE OF 1 AT THE I-TH
41.     00      C RESIDUAL POSITION. HOMOGENEOUS B.C.'S ARE ASSUMED EVERYWHERE,
42.     00      C INCLUDING THE TOP OF THE PRESENT SUBREGION NB.
43.     00      JL=JH
44.     00      GO TO 1DD
45.     00      END
46.     00
47.     00

```

VERSION 5
(continued)

```

1.      01      SUBROUTINE NORIAK,AY,BB,CX,RINV1,DUM0,DUM1,DUM2,F,H,X,IE,
2.      00      IMD,ND,M2,N2,NBLK,NBLK1
3.      00      DIMENSION AX(M2,N2),AY(M2,N2),BB(M2,N2),CX(M2,N2),RINV1(M2,M2,NBLK),
4.      00      A,RINV1(M2,M2,NBLK1),H(MD,ND),IE(NBLK1),DUM0(M2,NBLK1),DUM1(M2),
5.      00      B,DUM2(M2),F(M2,N2),X(MD,ND)
6.      00      JS=1
7.      00      DO 150 NB=1,NBLK
8.      00      JF=IE(NB)-2
9.      00      DO 105 J=JS,JF
10.     00      DO 105 I=1,M2
11.     00      1C5 X(I+1,J+2)=F(I,J)-AX(I,J)*X(I,J+1)-AY(I,J)*X(I+1,J)-BB(I,J)*
12.     00      A X(I+1,J+1)-CX(I,J)*X(I+2,J+1)
13.     00      IF (NB<ED,NBLK) GO TO 150
14.     00      J=IE(NB)-I
15.     00      DO 115 I=1,M2
16.     00      1C5 DUM1(I)=F(I,J)-AX(I,J)*X(I,J+1)-AY(I,J)*X(I+1,J)-BB(I,J)*
17.     00      A X(I+1,J+1)-CX(I,J)*X(I+2,J+1)-X(I+1,J+2)
18.     00      J=IE(NB)
19.     00      DO 120 M=1,M2
20.     00      DUM2(M)=DUM2(M)+DUM1(M)*RINV1(M,N,NB)
21.     00      DUM0(M,NB)=X(N,I,J)
22.     00      120 X(M+1,J)=X(M+1,J)-DUM2(M)
23.     00      150 JS=IE(NB)
24.     00      DO 300 NB1=1,NBLK
25.     00      NB=NBLK-NB1+I
26.     00      JS=I
27.     00      IF (NB>NE-1) JS=IE(NB-1)
28.     00      JF=IE(NB)-2
29.     00      IF (NB>ED,NBLK) GO TO 201
30.     00      J=IE(NB)
31.     00      DO 200 M=1,M2
32.     00      2C0 X(N+1,J)=DUM0(N,NB)
33.     00      2C1 N=IE(NB)
34.     00      DO 202 J=JS,N
35.     00      DO 202 I=I,MD
36.     00      2C2 H(I,J)=0.
37.     00      J=IE(NB)-1
38.     00      DO 210 I=1,M2
39.     00      210 DUM1(I)=F(I,J)-AX(I,J)*X(I,J+1)-AY(I,J)*X(I+1,J)-BB(I,J)*
40.     00      A X(I+1,J+1)-CX(I,J)*X(I+2,J+1)-X(I+1,J+2)
41.     00      DO 220 M=1,M2
42.     00      DUM2(M)=0.
43.     00      DO 220 M=1,M2
44.     00      220 H(M,NB)=DUM2(M)+DUM1(M)*RINV1(M,N,NB)
45.     00      DUM2(M)=0.
46.     00      DO 230 M=1,M2
47.     00      230 DUM2(M)=DUM2(M)+H(M+1,J5+1)*DUM2(M)
48.     00      J5=IE(NB-1)
49.     00      DO 240 M=1,M2
50.     00      DUM2(M)=0.
51.     00      DO 240 M=1,M2
52.     00      240 H(M+1,J)=DUM2(M)
53.     00      DO 250 J=JS,JF
54.     00      DO 250 I=I,M2
55.     00      250 H(I+1,J+2)=AX(I,J)*H(I,J+1)-AY(I,J)*H(I+1,J)-BB(I,J)*
56.     00      A H(I+1,J+1)-CX(I,J)*H(I+2,J+1)
57.     00      3C0 X(I+1,J+2)=X(I,J+2)+H(I+1,J+2)
58.     00      ENO

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VERSION 5
(continued)

```

1.      00      SUBROUTINE MATINV(B,N,M)
2.      00      DIMENSION B(N,1),B1(100),B2(100)
3.      00      M1=M-1
4.      00      DO 110 I=1,M1
5.      00      B1(I)=1./B(I,I)
6.      00      B(I,I)=1.0
7.      00      DO 112 J=1,M
8.      00      112 B(I,J)=B(I,J)*B1(I)
9.      00      IP1=I+1
10.     00      DO 120 II=IP1,M
11.     00      120 B1(II)=B(II,I)
12.     00      DO 125 II=IP1,M
13.     00      125 B(II,II)=0.
14.     00      DO 127 J=1,M
15.     00      127 B2(J)=B(I,J)
16.     00      DO 135 II=IP1,M
17.     00      DO 135 J=1,M
18.     00      135 B(II,J)=B(II,J)-B1(II)*B2(J)
19.     00      140 CONTINUE
20.     00      B1(I)=1./B(M,M)
21.     00      B(M,M)=1.
22.     00      DO 140 J=1,M
23.     00      140 B(M,J)=B(M,J)*B1(I)
24.     00      DO 150 I=2,M
25.     00      DO 155 I2=1,I
26.     00      155 B1(I2)=B(I2,I)
27.     00      IM1=I-1
28.     00      DO 156 I2=1,IM1
29.     00      156 B(I2,I)=0.
30.     00      DO 157 J=1,M
31.     00      157 B2(J)=B(I,J)
32.     00      IM1=I-1
33.     00      DO 160 I2=1,IM1
34.     00      DO 160 J=1,M
35.     00      160 B(I2,J)=B(I2,J)-B1(I2)*B2(J)
36.     00      150 CONTINUE
37.     00      END

```

END

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